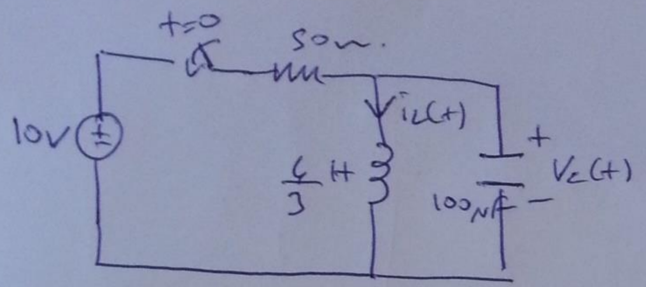
ex1



The circuit has no initial energy storage. find $i_L(t)$.
$\Downarrow$
inductor current.

Solution

This is a parallel RLC circuit.

$$\alpha = \frac{1}{2RC}, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\alpha = \frac{1}{2\times 50\times 100\times 10^{-6}} = 100\ s^{-1}$$

$$\omega_0 = \frac{1}{\sqrt{\frac{4}{3}\times 100\times 10^{-6}}} \cong 86.60 \text{ rad/sec.}$$

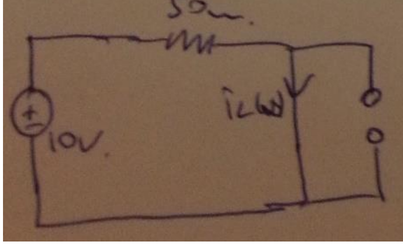
$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \cong -50$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \cong -150$$

$\alpha > \omega_0 \Rightarrow$ this is an overdamped circuit.

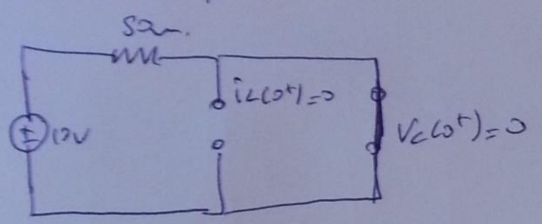
so $i_L(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3$

when $t \to \infty$, we have.



$$i_L(\infty) = \frac{10V}{50\Omega} = 0.2\,A. = \underline{\underline{A_3}}$$

for $t = 0^+$, we have.



$i_L(0^-) = i_L(0^+) = 0$

$V_C(0^-) = V_C(0^+) = 0$

$\Rightarrow$ The first Equation, for $t = 0^+$

$i_L(0^+) = 0 \Rightarrow 0 = A_1 e^0 + A_2 e^0 + 0.2.$

$$\boxed{A_1 + A_2 = -0.2}$$

The second Equation, for $t = 0^+$

$V_L = L \frac{di_L}{dt} \Rightarrow \frac{di_L(0^+)}{dt} = \frac{V_L(0^+)}{L}.$

$V_L(0^+) = V_C(0^+) = 0$

the voltage across inductor. The voltage across capacitor

$$\frac{di_L(t)}{dt} = \frac{d(A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3)}{dt}$$

$\frac{di_L(0^+)}{dt} = \frac{0}{\frac{4}{3}H.} = 0 = A_1 s_1 e^0 + A_2 s_2 e^0$

$\Rightarrow -50A_1 - 150A_2 = 0$

$\boxed{-A_1 - 3A_2 = 0} \rightarrow$ second Equ

$$\begin{array}{rl} \Rightarrow & A_1 + A_2 = -0.2 \\ + & -A_1 - 3A_2 = 0 \\ \hline & -2A_2 = -0.2 \end{array}$$

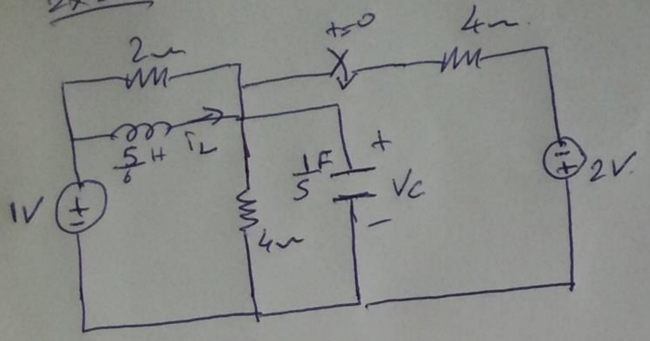
$\boxed{\begin{array}{l} A_2 = 0.1 \\ A_1 = -0.3 \end{array}} \Rightarrow$

$$\left(i_L(t) = -0.3e^{-50t} + 0.1e^{-150t} + 0.2\right) \quad t > 0$$

2

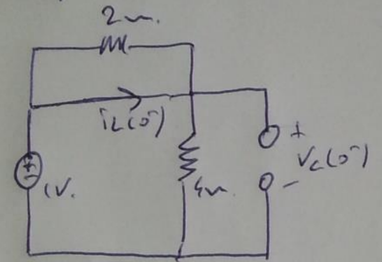
Ex 2

find $V_C(t)$.



Solution

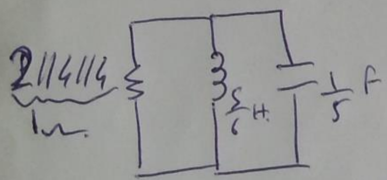
Before the switching, (for $t=0^-$)



$i_L(0^-) = \frac{1V}{4\Omega} = 0.25A$

$V_C(0^-) = 1V.$

⇒ the zero input circuit (to find the natural response part)



⇒ we can find α, w_0 and the roots (s_1, s_2) from this circuit

⇒ this is a parallel RLC circuit.

$\alpha = \frac{1}{2RC} = 2.5\ s^{-1}$, $w_0 = \frac{1}{\sqrt{LC}} = 2.45$ rad/s

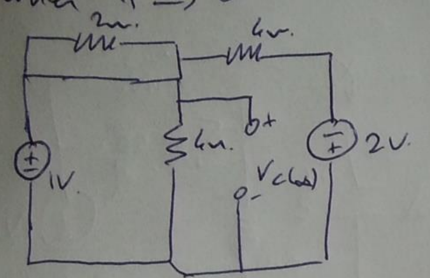
$s_1 = -\alpha + \sqrt{\alpha^2 - w_0^2} = -2$, $s_2 = -\alpha - \sqrt{\alpha^2 - w_0^2} = -3$

$\alpha > w_0$, so the circuit is overdamped.

$V_C(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3 \quad t > 0$

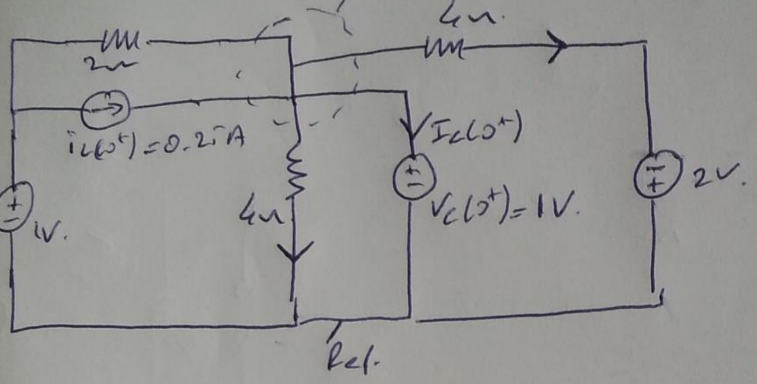
when $t \to \infty$ (to find forced response)

$V_c(\infty) = 1V.$ (parallel to 1V source)



at $t = 0^+$

→ KCL to this node. (node voltage = 1V) (parallel to $V_c(0^+)$).



First Equ.

for $t = 0^+ \Rightarrow V_c(0^+) = 1V = A_1 e^0 + A_2 e^0 + \overset{A_3}{1}$

$\Rightarrow \boxed{A_1 + A_2 = 0}$ → first Eq.

Second Equ.

$$KCL \Rightarrow 0.25 = \frac{1}{4} + \frac{(1-(-2))}{4} + I_c(0^+)$$

$$\Rightarrow I_c(0^+) = -\frac{3}{4} = -0.75A$$

⇒ we know that

$$I_c = C\frac{dV_c}{dt} \Rightarrow \frac{dV_c(0^+)}{dt} = \frac{I_c(0^+)}{C}$$

$$\Rightarrow \frac{-0.75}{\frac{1}{5}} = \frac{dV_c(0^+)}{dt} \Rightarrow \frac{dV_c(0^+)}{dt} = -3.75.$$

$$\Rightarrow \frac{dV_c(t)}{dt} = \frac{d(A_1 e^{\overset{-2}{s_1} t} + A_2 e^{\overset{-3}{s_2} t} + A_3)}{dt} = -2A_1 e^{-2\overset{0^+}{t}} - 3A_2 e^{-3\overset{0^+}{t}} + 0 \Rightarrow \text{for } t = 0^+$$

$\boxed{-2A_1 - 3A_2 = -3.75}$ → Second Eq.

2/ $A_1 + A_2 = 0$

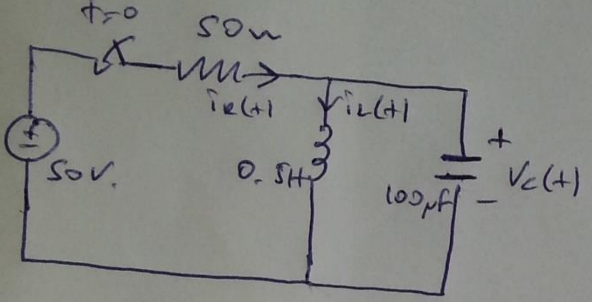
$+ \; -2A_1 - 3A_2 = -3.75$

$$-A_2 = -3.75$$

$$\boxed{\begin{aligned} A_2 &= 3.75 \\ A_1 &= -3.75 \end{aligned}}$$

then $v_c(t) = \left(-3.75\,e^{-2t} + 3.75\,e^{-3t} + 1\right) \quad t > 0$

Ex3



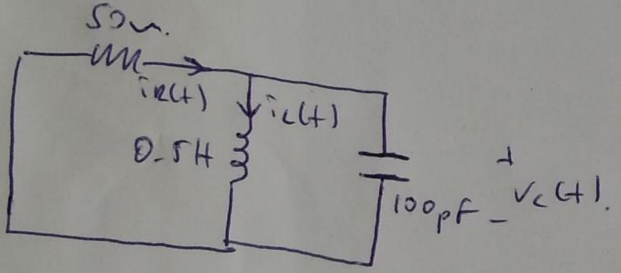
$i_L(0^-) = 2A$

$V_C(0^-) = 0V.$

Find $i_R(t)$

Solution

Zero input circuit



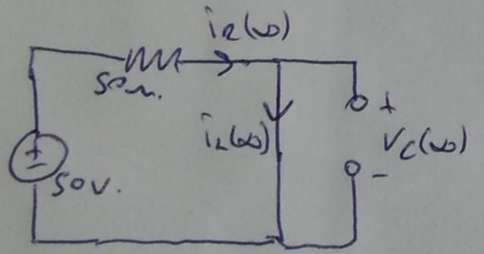
⇒ This is a parallel RLC circuit.

$\Rightarrow \alpha = \frac{1}{2RC} = 100\ s^{-1}$, $w_0 = \frac{1}{\sqrt{LC}} = 141.4\ rad/s.$

$w_0 > \alpha$ ⇒ so this circuit is overdamped.

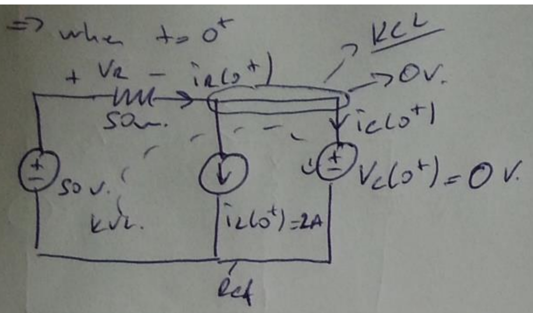
$i_R(t) = e^{-\alpha t}[A_1\cos(w_d t) + A_2\sin(w_d t)] + A_3$

⇒ In order to find A_3, we need to find $i_R(\infty)$ when $t \to \infty$



$i_R(\infty) = \frac{50V}{50\Omega} = \boxed{1V. = A_3}$

$\Rightarrow$ when $t = 0^+$



$$i_R(0^+) = \frac{(50 - 0)V}{50\Omega} = 1A$$

The first equation, when $t = 0^+$

$$i_R(0^+) = 1A = e^0\left[A_1\underbrace{\cos 0}_{1} + \underbrace{A_2\sin 0}_{0}\right] + \underset{1A}{A_3}$$

$$\Rightarrow 1A = A_1 + 1 \Rightarrow \boxed{A_1 = 0}$$

The second equation, (we need to find the derivative of $i_R(0^+)$)

$$I_C = C\frac{dV_C}{dt} \Rightarrow \frac{dV_C}{dt} = \frac{I_C}{C}$$

when we apply KCL, for $t = 0^+$

$$\underbrace{\frac{50 - 0}{50}}_{1} = 2A + i_C(0^+) \Rightarrow \boxed{i_C(0^+) = -1A}$$

$$\Rightarrow \frac{dV_C(0^+)}{dt} = \frac{-1}{100\times 10^{-6}} = -10.000$$

$\Rightarrow$ if we write $V_C(0^+)$ in terms of $i_R(0^+)$.

KVL $\Rightarrow$ $-50 + V_R(0^+) + V_C(0^+) = 0$

$$V_C(0^+) = 50 - V_R(0^+) \Rightarrow V_C(0^+) = \boxed{50 - i_R(0^+)\times 50}$$

↳ if we use this equation instead of $V_C(0^+)$.

(2)

$$\frac{d(V_c(0^+))}{dt} = -10000$$

$$\Rightarrow \frac{d(50 - 50\, i_R(0^+))}{dt} = -10000$$

$$\Rightarrow -50 \frac{d i_R(0^+)}{dt} = -10000$$

$$\frac{d i_R(0^+)}{dt} = \frac{\cancel{+}1000\cancel{0}}{\cancel{+}5\cancel{0}} = 200$$

the second Equation.

for $t = 0^+$

$$-\alpha e^{-\alpha t}[A_1 \cos(w_d t) + A_2 \sin(w_d t)] + e^{-\alpha t}[-A_1 w_d \sin(w_d t) + A_2 w_d \cos(w_d t)]$$

$$\Rightarrow -100 e^{-100t}[A_2 \sin(w_d t)] + e^{-100t}[A_2 w_d \cos(w_d t)]$$

for $t \Rightarrow 0$.

$$-100 e^0[\underbrace{A_2 \sin 0}_{0}] + e^0[A_2 w_d \underbrace{\cos 0}_{1}] = 200$$

$w_d = \sqrt{w_0^2 - \alpha^2} \cong 100$ rad/sec.

$\Rightarrow$

$$A_2 100 = 200$$

$$\boxed{A_2 = \frac{200}{100} = 2}$$

So the find formula of $i_R(t)$.

$$e^{-100t}[2\sin(100t)] + 1 = i_R(t) \quad \text{for } t > 0$$

$$i_R(t) = 2e^{-100t}\sin(100t) + 1 \text{ A} \quad \text{for } t > 0$$