BME2301 - Circuit Theory

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Objectives of the Lecture

- Present Kirchhoff's Current and Voltage Laws.
- Demonstrate how these laws can be used to find currents and voltages in a circuit.
- Explain how these laws can be used in conjunction with Ohm's Law.

Important Note : Laboratory Sections of the course will start in the 3rd week. Follow Nihat Akkan's Avesis Web Page.

Resistivity, ρ

- Resistivity is a material property
  - Dependent on the number of free or mobile charges (usually electrons) in the material.
    - In a metal, this is the number of electrons from the outer shell that are ionized and become part of the 'sea of electrons'
  - Dependent on the mobility of the charges
    - Mobility is related to the velocity of the charges.
    - It is a function of the material, the frequency and magnitude of the voltage applied to make the charges move, and temperature.

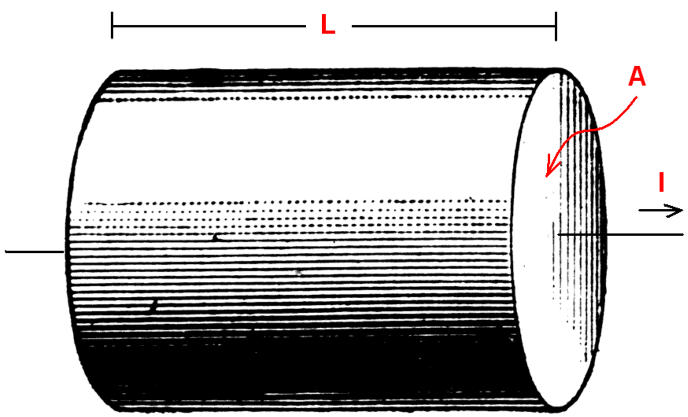
Resistivity of Common Materials at Room Temperature (300K)

Material	Resistivity (Ω-cm)	Usage
Silver	1.64×10^{-8}	Conductor
Copper	1.72×10^{-8}	Conductor
Aluminum	2.8×10^{-8}	Conductor
Gold	2.45×10^{-8}	Conductor
Carbon (Graphite)	4×10^{-5}	Conductor
Germanium	0.47	Semiconductor
Silicon	640	Semiconductor
Paper	10^{10}	Insulator
Mica	5×10^{11}	Insulator
Glass	10^{12}	Insulator
Teflon	3×10^{12}	Insulator

Resistance, R

- Resistance takes into account the physical dimensions of the material

$$R = \rho \frac{L}{A}$$



- where:
  - L is the length along which the carriers are moving
  - A is the cross sectional areathat the free charges move through.

Ohm's Law

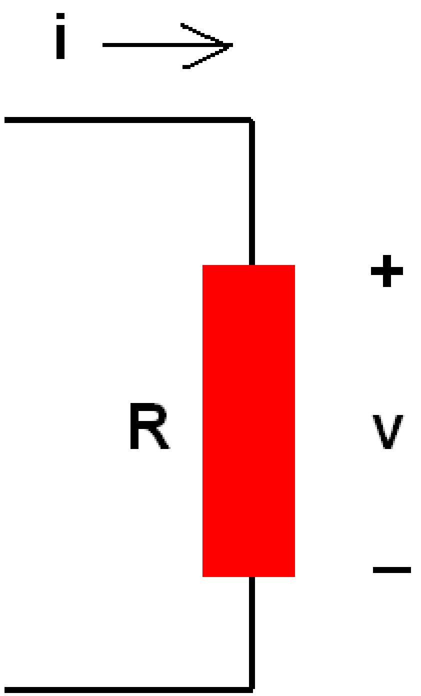
- Voltage drop across a resistor is proportional to the current flowing through the resistor

$$\mathrm{V} = iR$$

Units: $V = A\Omega$

where $A = C/s$

Short Circuit



- If the resistor is a perfect conductor (or a short circuit)

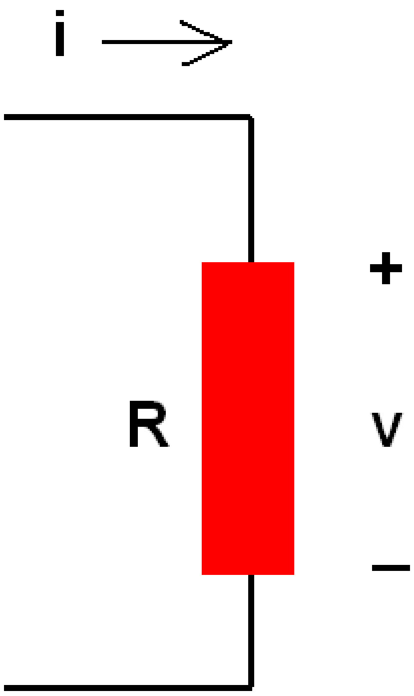
  $R = 0\ \Omega$,

- then

  $v = iR = 0\ V$

  – no matter how much current is flowing through the resistor

Open Circuit



- If the resistor is a perfect insulator, $R = \infty\ \Omega$

- then

$$i = \lim_{R \to \infty} \frac{v}{R} = 0$$

  – no matter how much voltage is applied to (or dropped across) the resistor.

Conductance, G

- Conductance is the reciprocal of resistance

$G = R^{-1} = i/v$

– Unit for conductance is S (siemens) or (mhos, ℧)

$G = A\sigma/L$

where σ is conductivity,

which is the inverse of resistivity, ρ

Power Dissipated by a Resistor

$$p = iv = i(iR) = i^2R$$

$$p = iv = (v/R)v = v^2/R$$

$$p = iv = i(i/G) = i^2/G$$

$$p = iv = (vG)v = v^2G$$

Power (con't)

- Since R and G are always real positive numbers
  - Power dissipated by a resistor is always positive
- The power consumed by the resistor is not linear with respect to either the current flowing through the resistor or the voltage dropped across the resistor
  - This power is released as heat. Thus, resistors get hot as they absorb power (or dissipate power) from the circuit.

Short and Open Circuits

- There is no power dissipated in a short circuit.

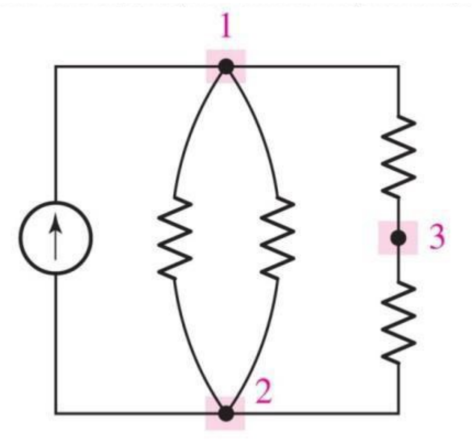
$$p_{sc} = \mathrm{v}^2 R = (0\mathrm{V})^2 (0\Omega) = 0\mathrm{W}$$

- There is no power dissipated in an open circuit.

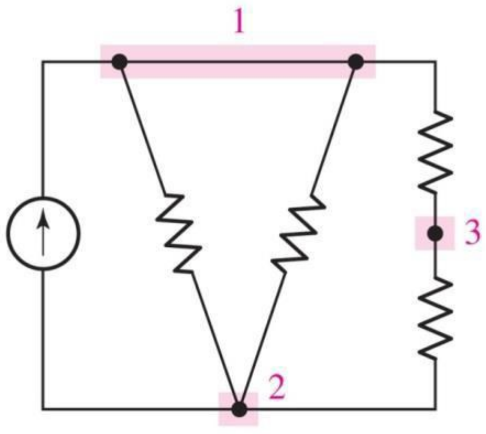
$$p_{oc} = \mathrm{i}^2 / R = (0\mathrm{A})^2 / (\infty\,\Omega) = 0\mathrm{W}$$

Circuit Terminology

- Node
  - point at which 2+ elements have a common connection
    - e.g., node 1, node 2, node 3
- Path
  - a route through a network, through nodes that never repeat
    - e.g., 1→3→2, 1→2→3
- Loop
  - a path that starts & ends on the same node
    - e.g., 3→1→2→3
- Branch
  - a single path in a network; contains one element and the nodes at the 2 ends
    - e.g., 1→2, 1→3, 3→2



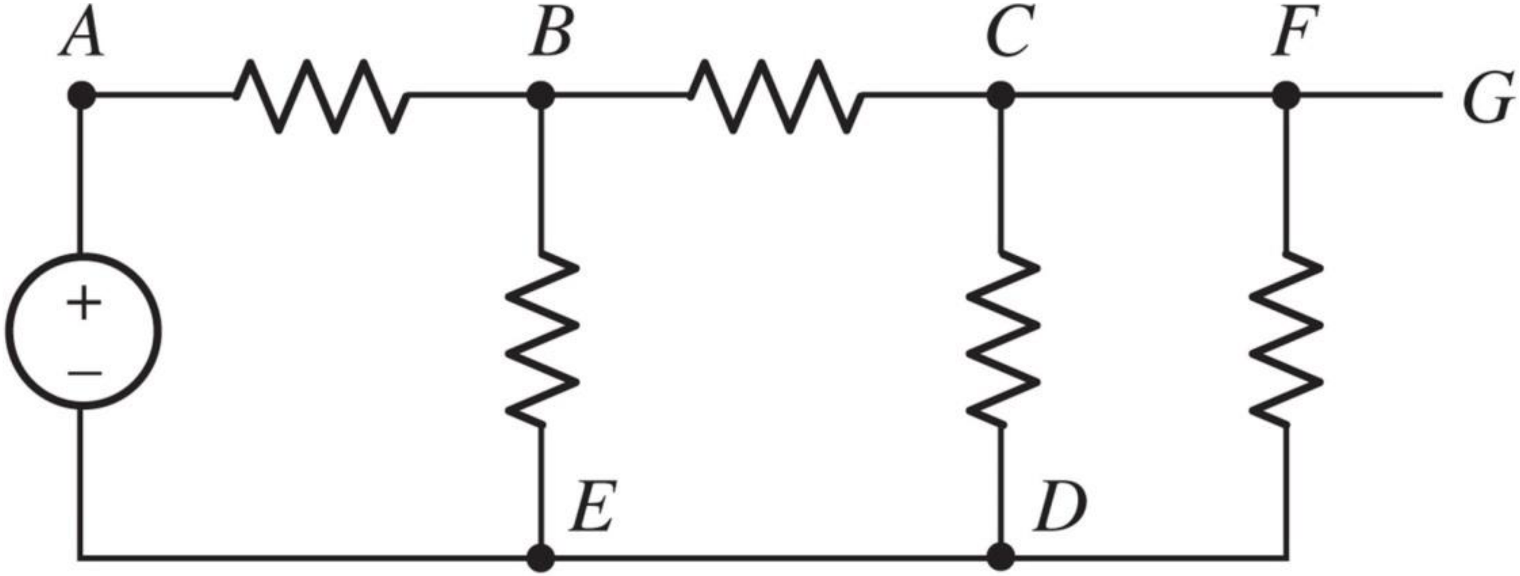
(a)



(b)

Exercise

- For the circuit below:
  a. Count the number of circuit elements.
  b. If we move from *B* to *C* to *D*, have we formed a path and/or a loop?
  c. If we move from *E* to *D* to *C* to *B* to *E*, have we formed a path and/or a loop?



Kirchhoff's Current Law (KCL)

- Gustav Robert Kirchhoff: German university professor, born while Ohm was experimenting
- Based upon conservation of charge
  - the algebraic sum of the charge within a system can not change.
  - the algebraic sum of the currents entering any node is zero.

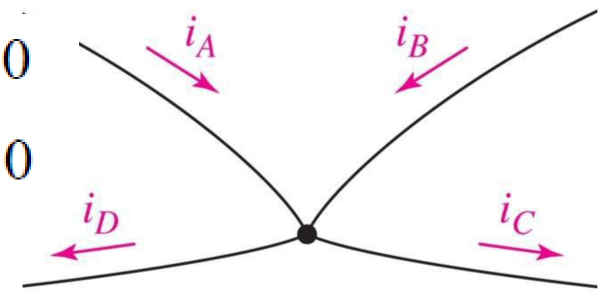
$$\sum_{n=1}^{N} i_n = 0$$

Where N is the total number of branches connected to a node.

$$\sum_{\text{node}} i_{enter} = \sum_{\text{node}} i_{leave}$$

$$i_A + i_B - i_C - i_D = 0$$

$$-i_A - i_B + i_C + i_D = 0$$



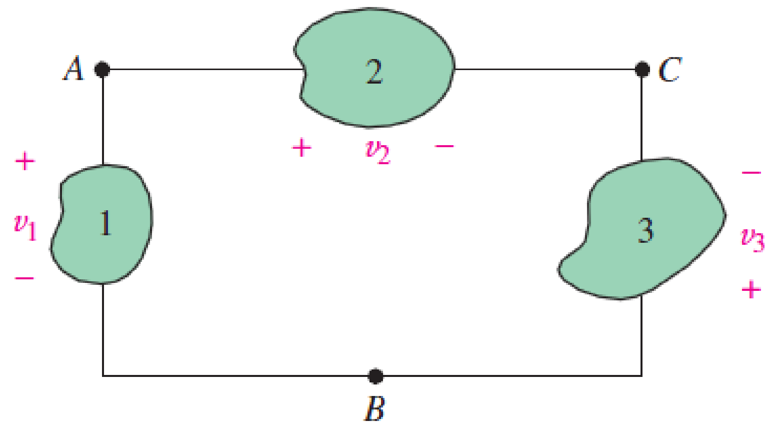
Kirchhoff's Voltage Law (KVL)

- Based upon conservation of energy
  - the algebraic sum of voltages dropped across components around a loop is zero.
  - The energy required to move a charge from point A to point B must have a value independent of the path chosen.

$$\sum_{m=1}^{M} v = 0$$

Where M is the total number of branches in the loop.

$$\sum v_{drops} = \sum v_{rises}$$

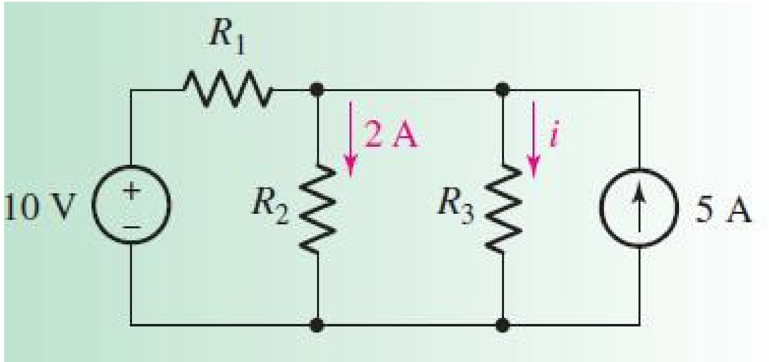


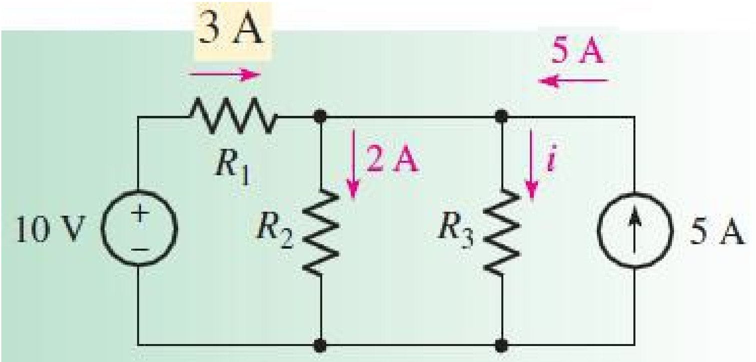
$$-v_1 + v_2 - v_3 = 0$$

$$v_1 - v_2 + v_3 = 0$$

Example-01

- For the circuit, compute the current through R_3 if it is known that the voltage source supplies a current of 3 A.
- Use KCL



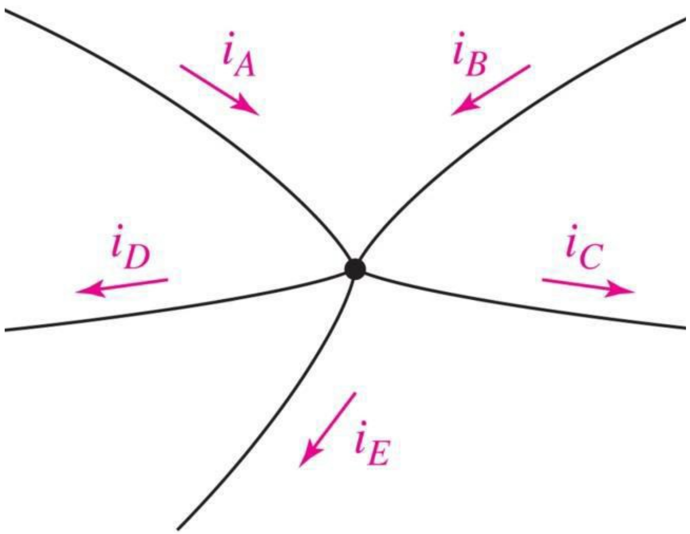


$$3 - 2 - i + 5 = 0$$

$$i = 3 - 2 + 5 = 6\text{ A}$$

Example-02

- Referring to the single node below, compute:
  a. i_B, given $i_A = 1$ A, $i_D = -2$ A, $i_C = 3$ A, and $i_E = 4$ A
  b. i_E, given $i_A = -1$ A, $i_B = -1$ A, $i_C = -1$ A, and $i_D = -1$ A



- Use KCL

$$i_A + i_B - i_C - i_D - i_E = 0$$

a. $i_B = -i_A + i_C + i_D + i_E$

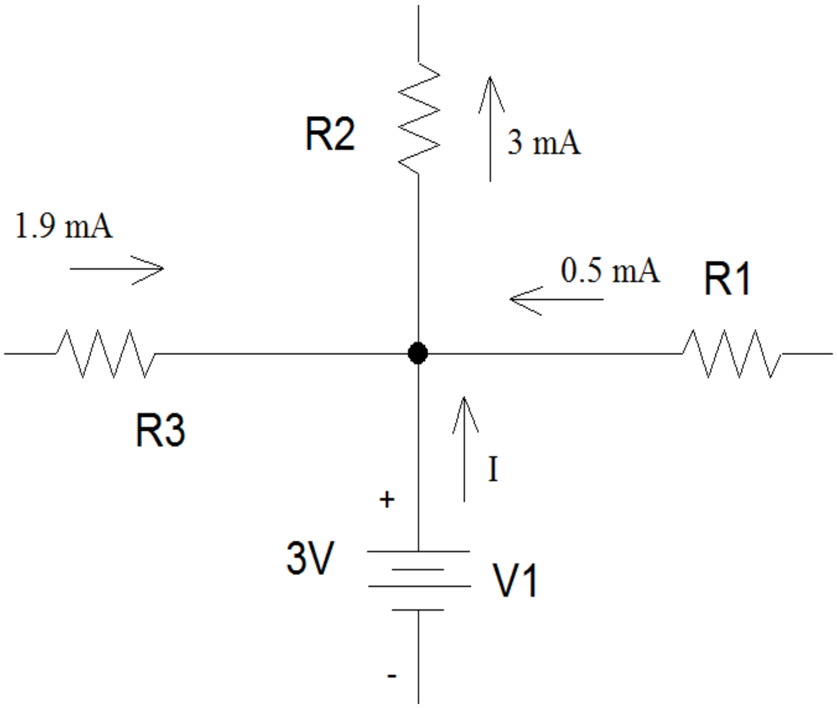
$i_B = -1 + 3 - 2 + 4 = 4$ A

b. $i_E = i_A + i_B - i_C - i_D$

$i_E = -1 - 1 + 1 + 1 = 0$ A

Example-03

- Determine I, the current flowing out of the voltage source.



– Use KCL
  - 1.9 mA + 0.5 mA + I are entering the node.
  - 3 mA is leaving the node.

$$1.9mA + 0.5mA + I = 3mA$$

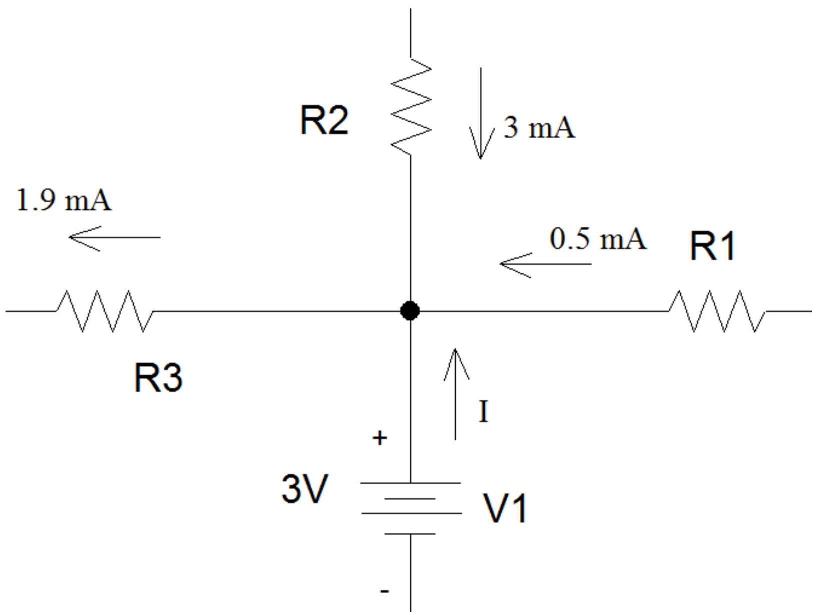
$$I = 3mA - (1.9mA + 0.5mA)$$

$$I = 0.6mA$$

V1 is generating power.

Example-04

- Suppose the current through R2 was entering the node and the current through R3 was leaving the node.



- Use KCL
  - 3 mA + 0.5 mA + I are entering the node.
  - 1.9 mA is leaving the node.

$$3mA + 0.5mA + I = 1.9mA$$

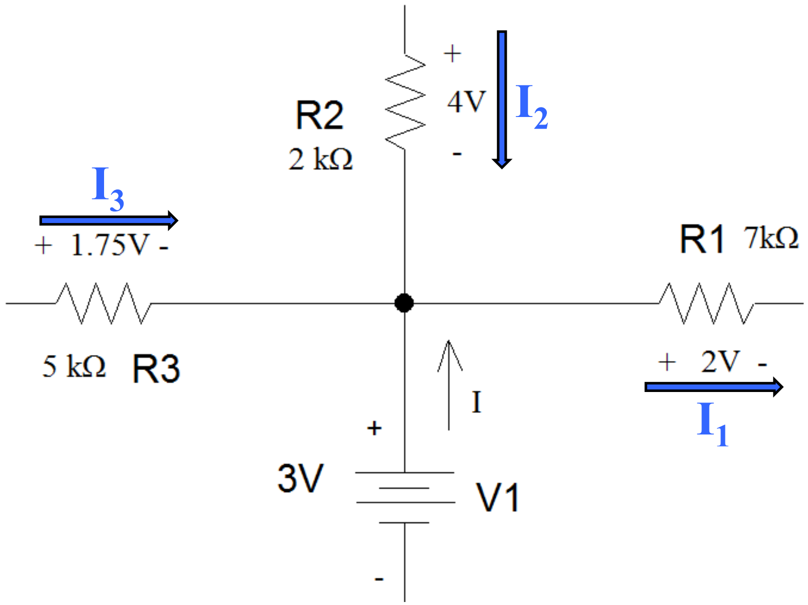
$$I = 1.9mA - (3mA + 0.5mA)$$

$$I = -1.6mA$$

V1 is dissipating power.

Example-05

- If voltage drops are given instead of currents,
  - you need to apply Ohm's Law to determine the current flowing through each of the resistors before you can find the current flowing out of the voltage supply.
    - I_1 is leaving the node.
    - I_2 is entering the node.
    - I_3 is entering the node.
    - I is entering the node.

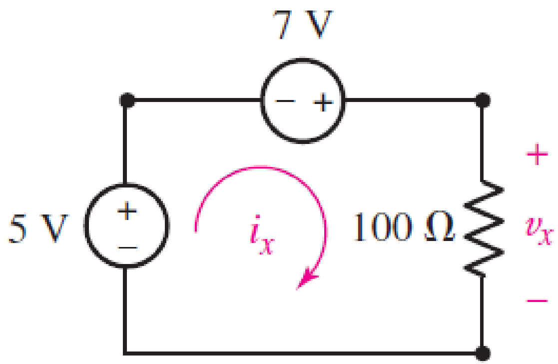


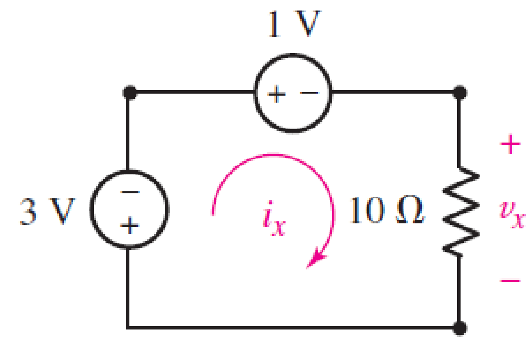
$$I_1 = 2V / 7k\Omega = 0.286mA$$
$$I_2 = 4V / 2k\Omega = 2mA$$
$$I_3 = 1.75V / 5k\Omega = 0.35mA$$

$$I_2 + I_3 + I = I_1$$
$$2mA + 0.35mA + I = 0.286mA$$
$$I = 0.286mA - 2.35mA = -2.06mA$$

Example-06

- For each of the circuits in the figure below, determine the voltage v_x and the current i_x.





  – Applying KVL clockwise around the loop and Ohm's law

$$-5 - 7 + v_x = 0$$

$$v_x = 12 \text{ V}$$

$$i_x = \frac{v_x}{100} = \frac{12}{100} \text{ A} = 120 \text{ mA}$$

$$+3 + 1 + v_x = 0$$

$$v_x = \underline{-4 \text{ V}}$$

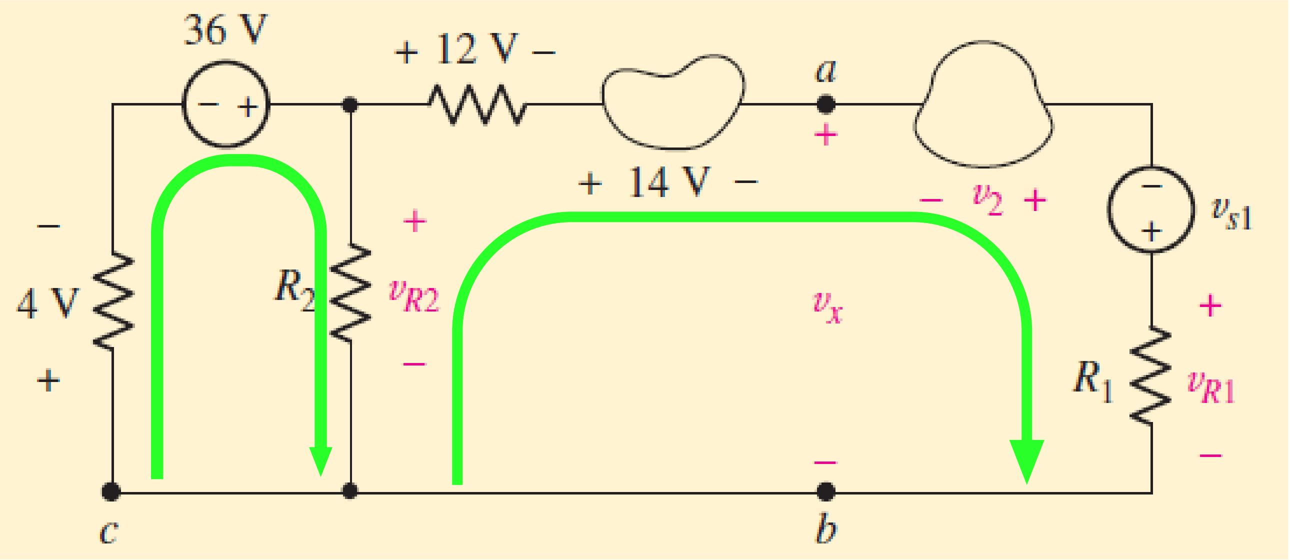
$$i_x = \frac{v_x}{10} = \underline{-400 \text{ mA}}$$

Example-07

- For the circuit below, determine

a. v_{R2}

b. v_x



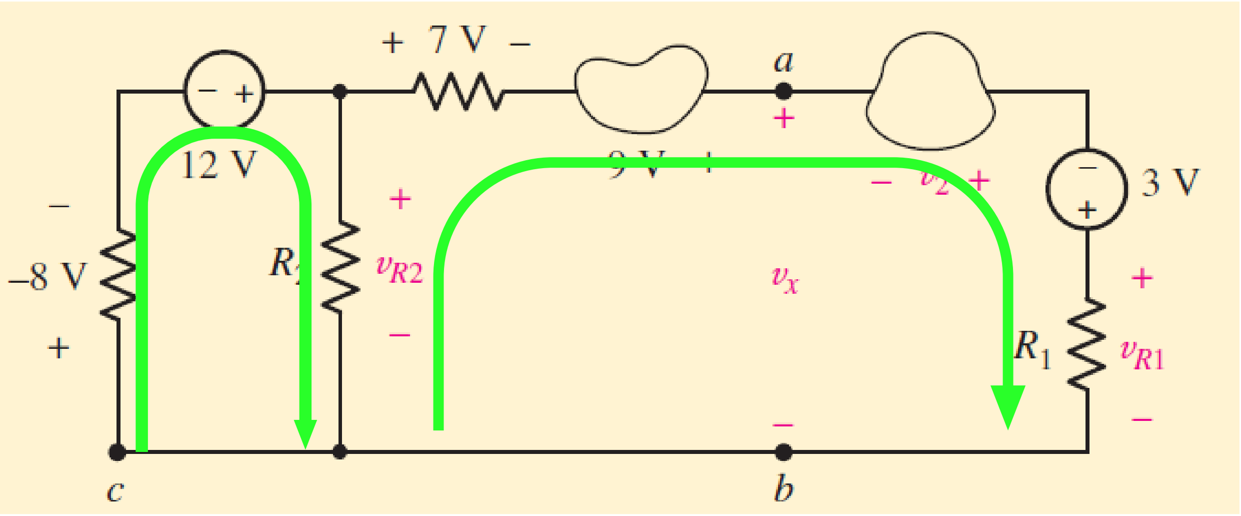
a. $4 - 36 + v_{R2} = 0$ $\quad$ $v_{R2} = 32$ V

b. $-32 + 12 + 14 + v_x = 0$ $\quad$ $v_x = 6$ V

Example-08

- For the circuit below, determine

  a. v_{R2} b. v_x if $v_{R1} = 1$ V.



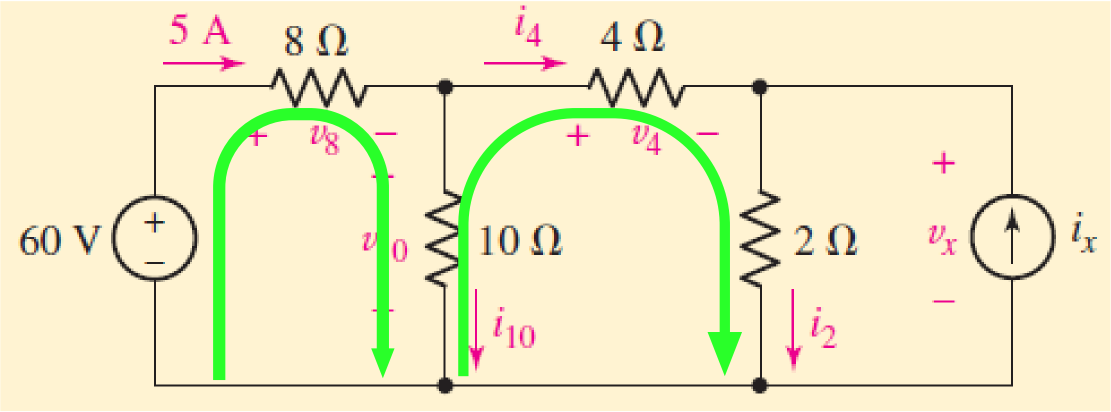
a. KVL yields $-8 - 12 + v_{R2} = 0$ $\underline{v_{R2} = 20 \text{ V}}$

b. KVL yields $-20 + 7 - 9 - v_2 - 3 + v_{R1}$

where $v_{R1} = 1$ V. Thus, $\underline{v_2 = -24 \text{ V}}$

Example-09

- For the circuit below, determine v_x



$$-60 + v_8 + v_{10} = 0$$

$$v_{10} = 0 + 60 - 40 = 20 \text{ V}$$

$$-v_{10} + v_4 + v_x = 0$$

$$v_x = 20 - v_4$$

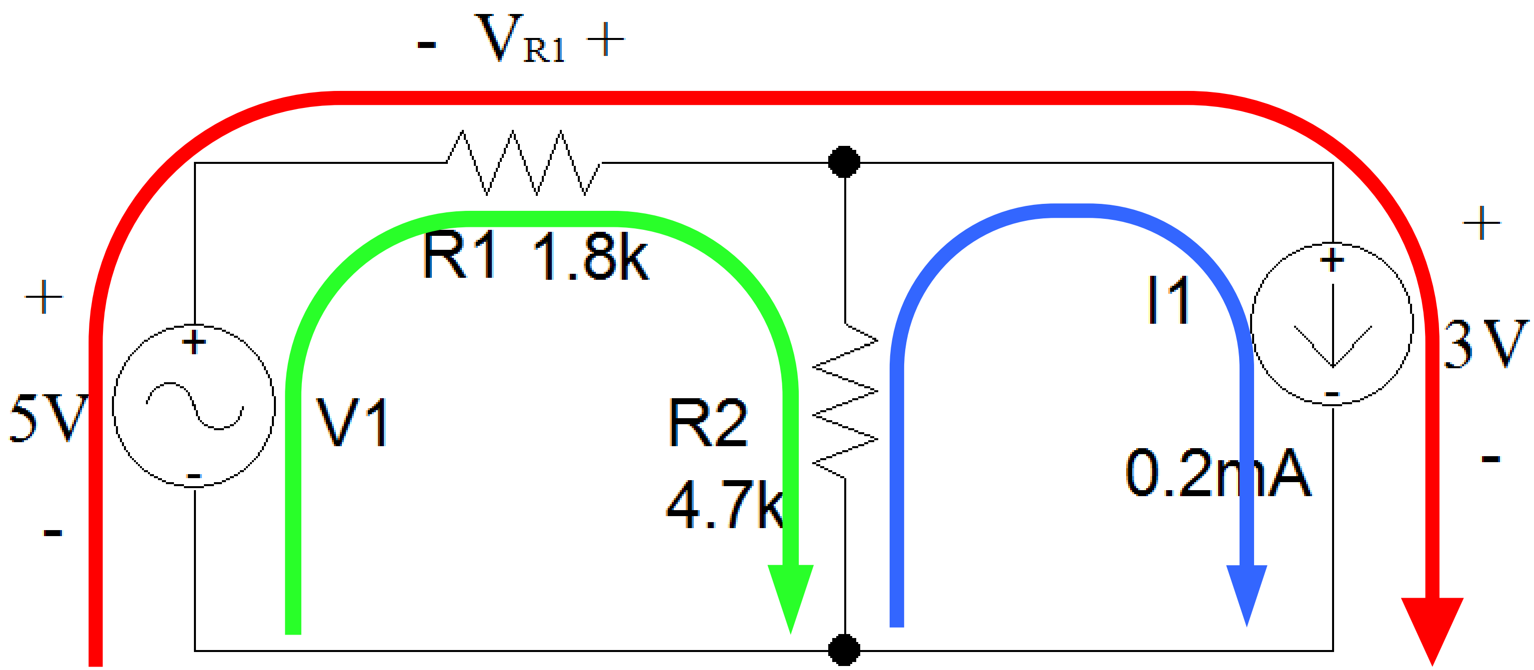
$$i_4 = 5 - i_{10} = 5 - \frac{v_{10}}{10} = 5 - \frac{20}{10} = 3$$

$$v_4 = (4)(3) = 12 \text{ V}$$

$$v_x = 20 - 12 = 8 \text{ V}$$

Example-10…

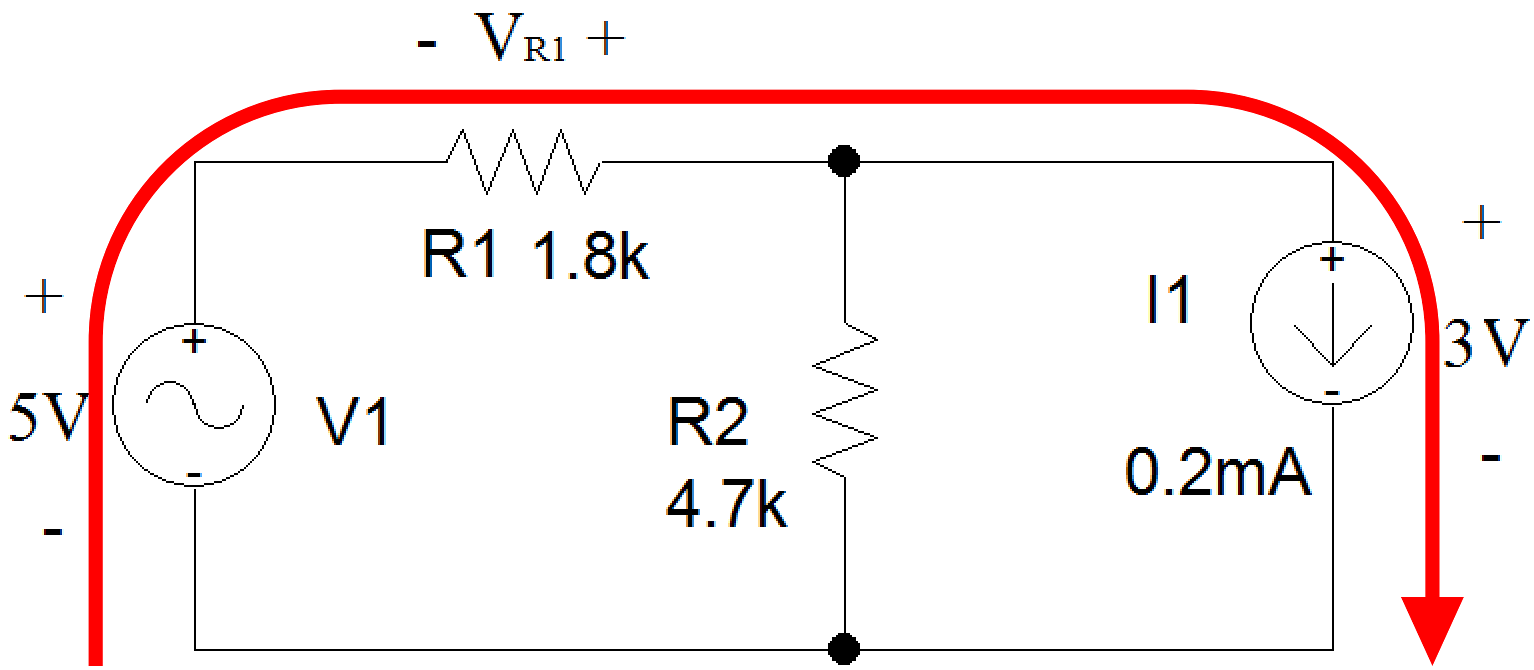
- Find the voltage across R1.
  - Note that the polarity of the voltage has been assigned in the circuit schematic.



– First, define a loop that include R1.

…Example-10…

- If the red loop is considered

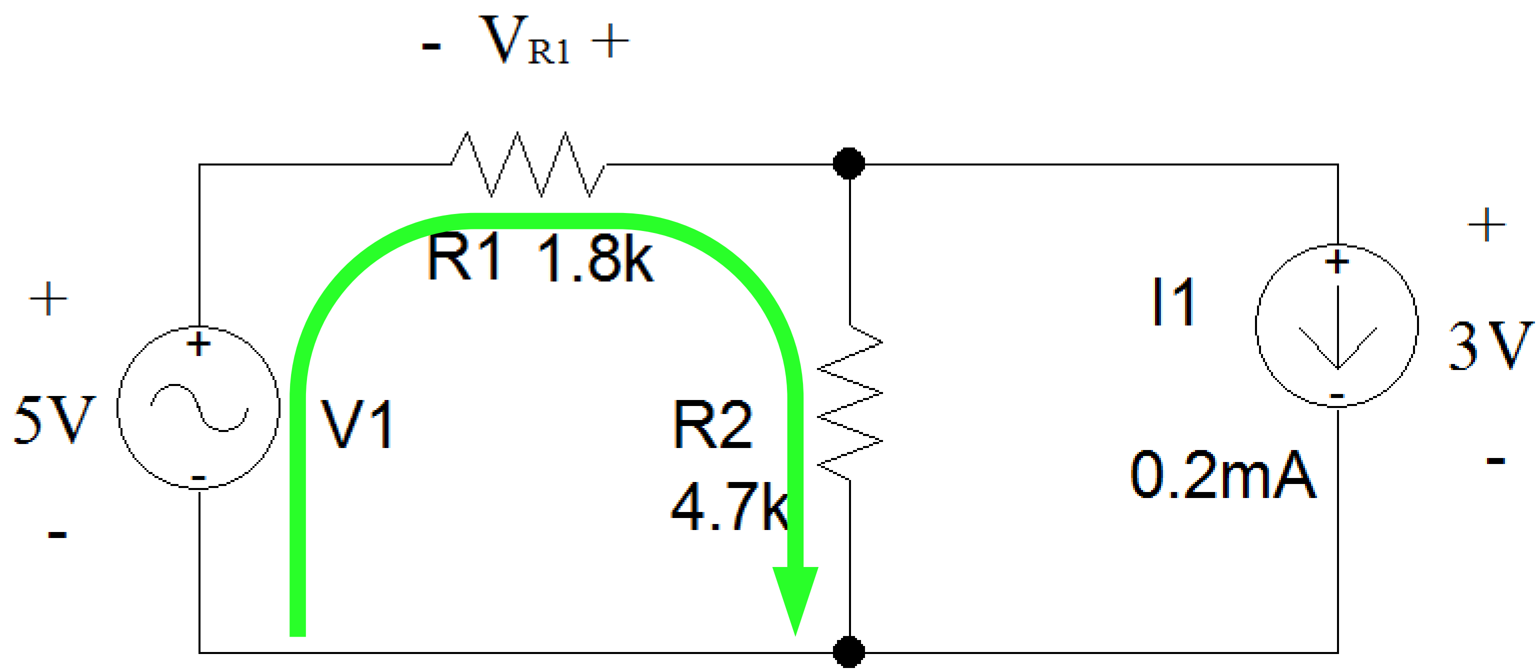


  – By convention, voltage drops are added and voltage rises are subtracted in KVL.

$$-5\text{ V} - V_{R1} + 3\text{ V} = 0 \qquad\qquad V_{R1} = 2\text{ V}$$

…Example-10

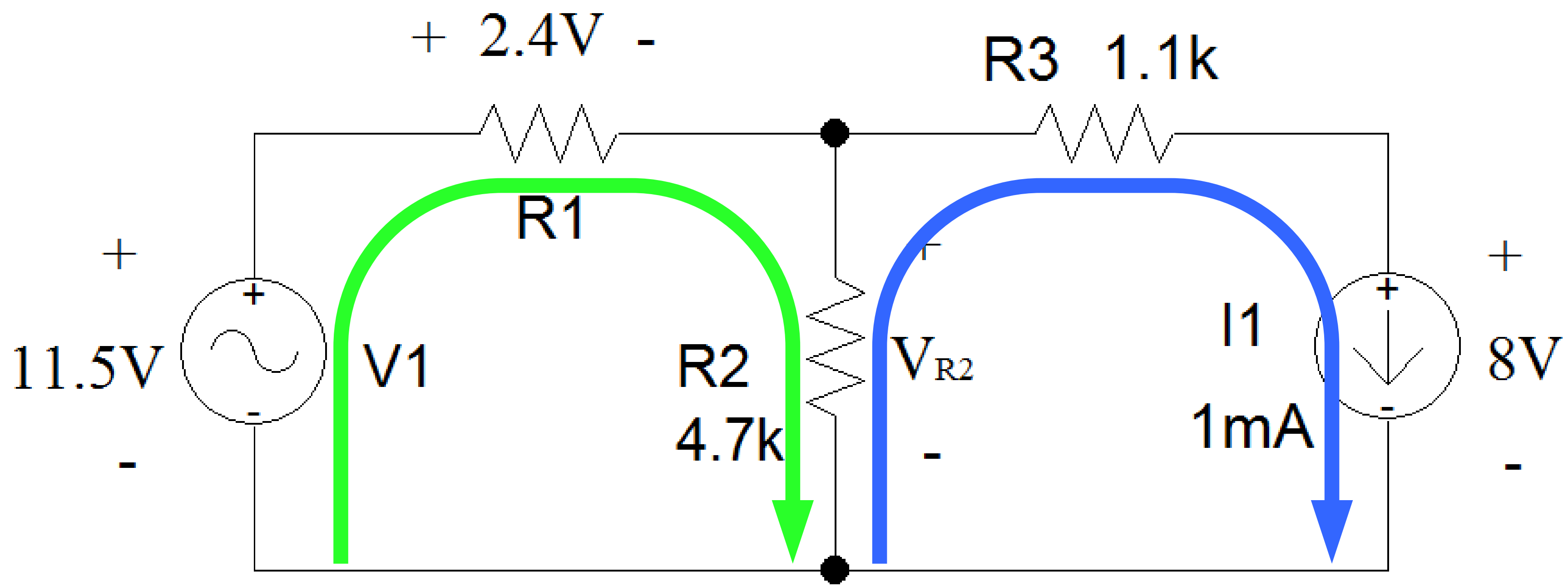
- Suppose you chose the green loop instead.
  - Since R2 is in parallel with I1, the voltage drop across R2 is also 3V.



$-5\text{ V} - V_{R1} + 3\text{ V} = 0$ $\quad$ $V_{R1} = 2\text{ V}$

Example-11…

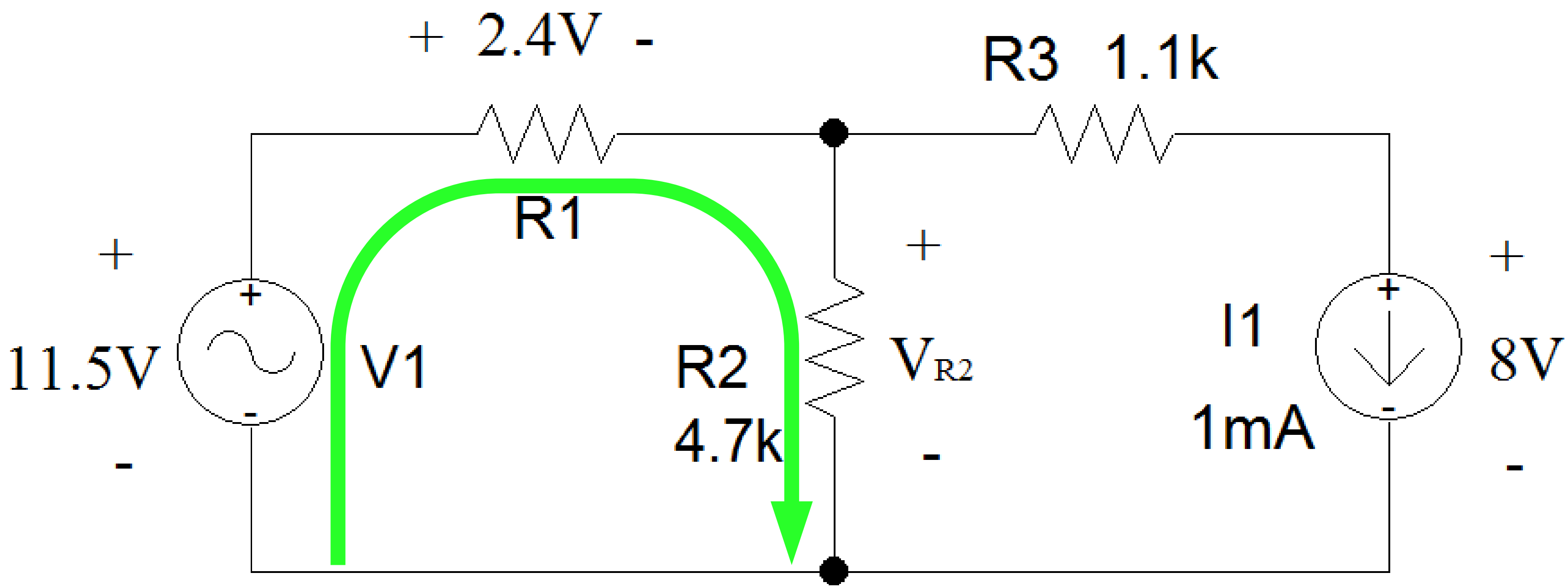
- Find the voltage across R2 and the current flowing through it.



  - First, draw a loop that includes R2.

…Example-11…

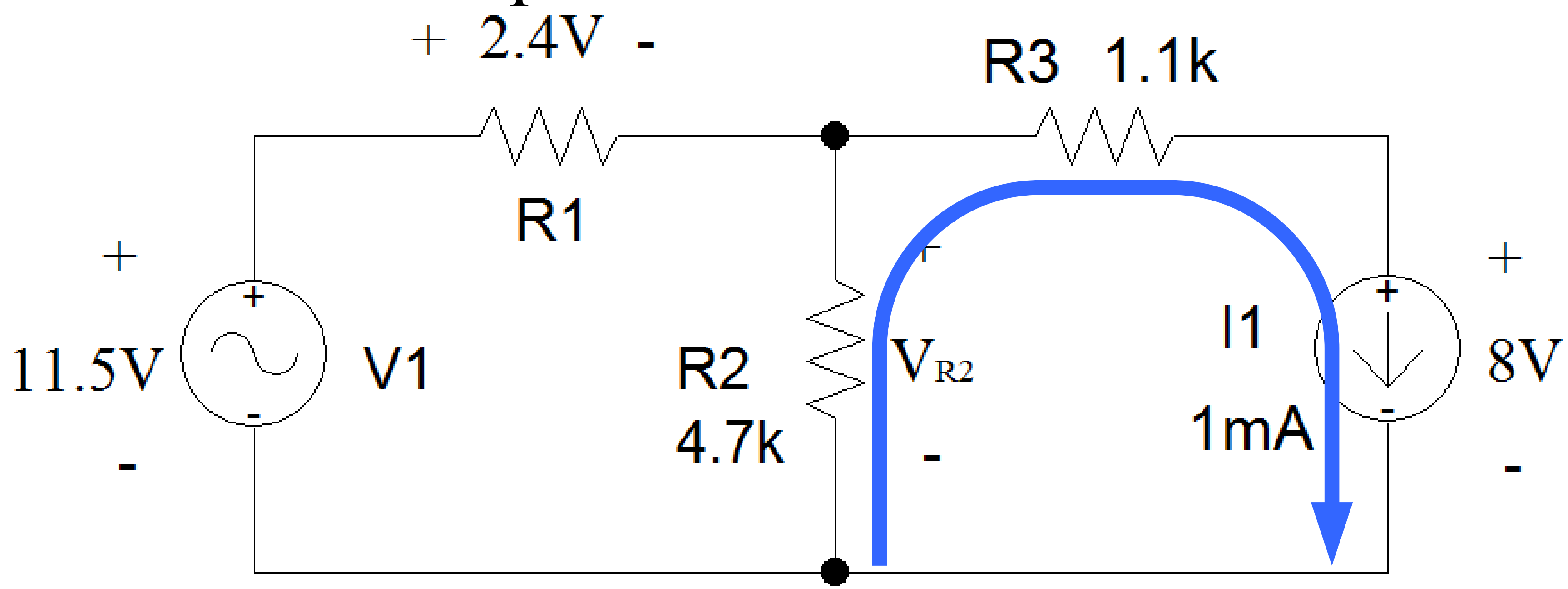
- If the green loop is used:



$-11.5\ V + 2.4\ V + V_{R2} = 0$ $\qquad$ $V_{R2} = 9.1\ V$

…Example-11…

- If the blue loop is used:



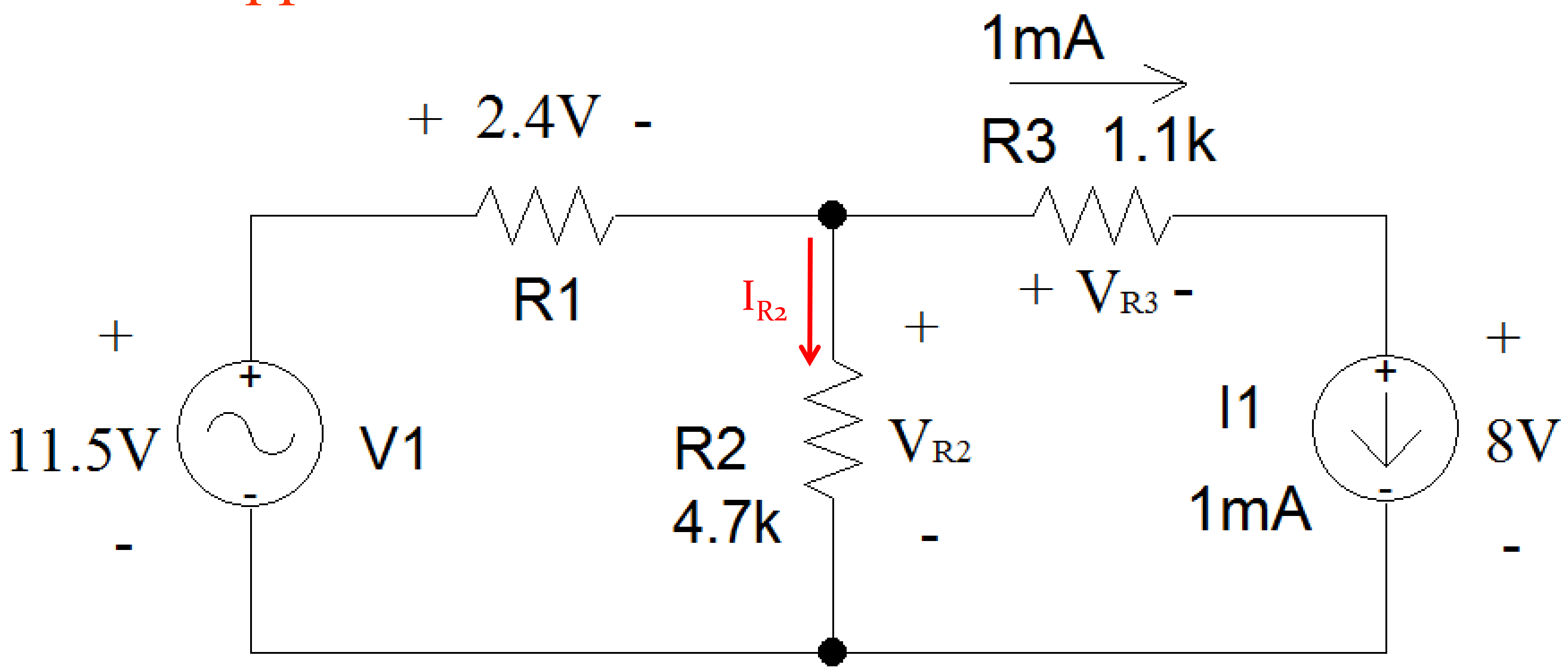
- First, find the voltage drop across R3

$1 \text{ mA} \times 1.1 \text{ k}\Omega = 1\times10^{-3} \text{ A} \times 1.1\times10^{3} \ \Omega = 1.1 \text{ V}$

$1.1 \text{ V} + 8 \text{ V} - V_{R2} = 0 \qquad V_{R2} = 9.1 \text{ V}$

…Example-11

– Once the voltage across R2 is known, Ohm's Law is applied to determine the current.



$I_{R2} = 9.1\text{ V} / 4.7\text{ k}\Omega = 9.1\text{ V} / (4.7 \times 10^{3}\ \Omega)$

$I_{R2} = 1.94 \times 10^{-3}\text{ A} = 1.94\text{ mA}$