

# BME2301 - Circuit Theory

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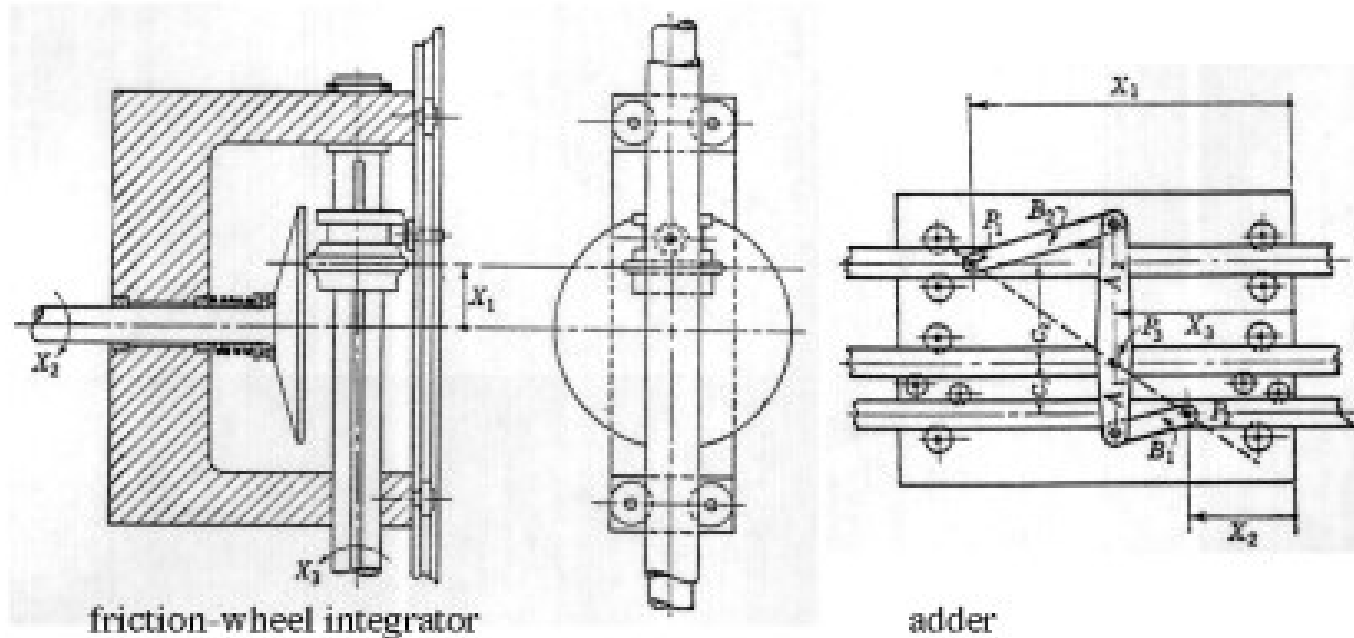
<https://avesis.yildiz.edu.tr/nakkan>

# **1<sup>st</sup> Order Op Amp Circuits**

# Objectives of Lecture

- Discuss analog computing and the application of 1<sup>st</sup> order operational amplifier circuits.
- Derive the equations that relate the output voltage to the input voltage for a differentiator and integrator.
- Explain the source of the phase shift between the output and input voltages.

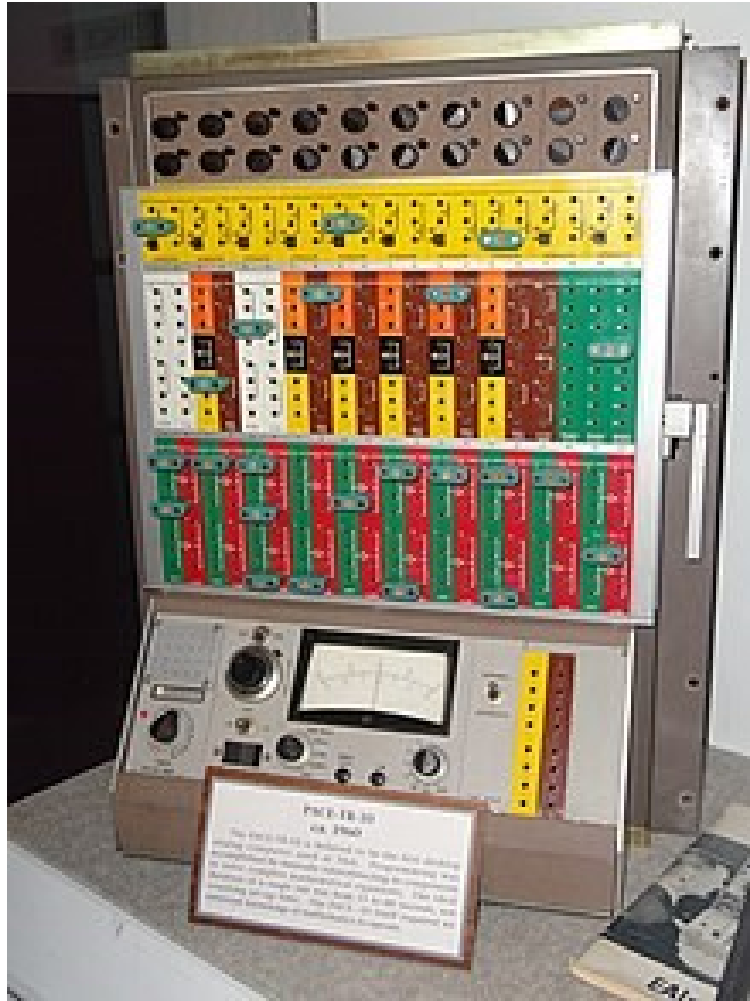
# Mechanical Analog Computers



Designed by Vannevar Bush in 1930 and used to control position of artillery through WWII. Replaced by electrical analog computers towards the end of WWII, which performed the needed calculations much faster.

<http://www.science.uva.nl/museum/AnalogComputers.html>

# Analog computers



- a type of computer that uses the continuously changeable aspects of physical phenomena such as electrical, mechanical, or hydraulic quantities to model the problem being solved.
- Analog computers were widely used in scientific and industrial applications where digital computers of the time lacked sufficient performance.
- Analog computers can have a very wide range of complexity.

# Why Use an Analog Computer?

- Calculations performed in real time without the use of a 'real' computer.
  - Can be integrated into the instrumentation circuitry.
    - Commonly used in control circuits to rapidly monitor and change conditions without the need to communicate back and forth with a digital computer.
    - Power consumption is not high.
- Input can be any value between  $V^+$  and  $V^-$ .
  - Can be designed to handle large (or small) voltages.
  - No digitizing errors.

# Disadvantage

- Slow
  - Maximum frequency is less than 10 MHz
    - Compare this to the clock speed of your digital computer.
  - Voltage transfer function is nonlinear over entire range of input voltages.
  - Timing of inputs needs to be carefully considered.
    - Any time delays can cause errors in the calculations performed.

# Subsystems

- Multipliers
    - Inverting and non-inverting amplifiers
      - Typically fixed number, which means fixed resistor values in amplifiers
  - Adders and Subtractors
    - Summing and difference amplifiers
  - Differentiators
  - Integrators
- } 1<sup>st</sup> order op amp circuits

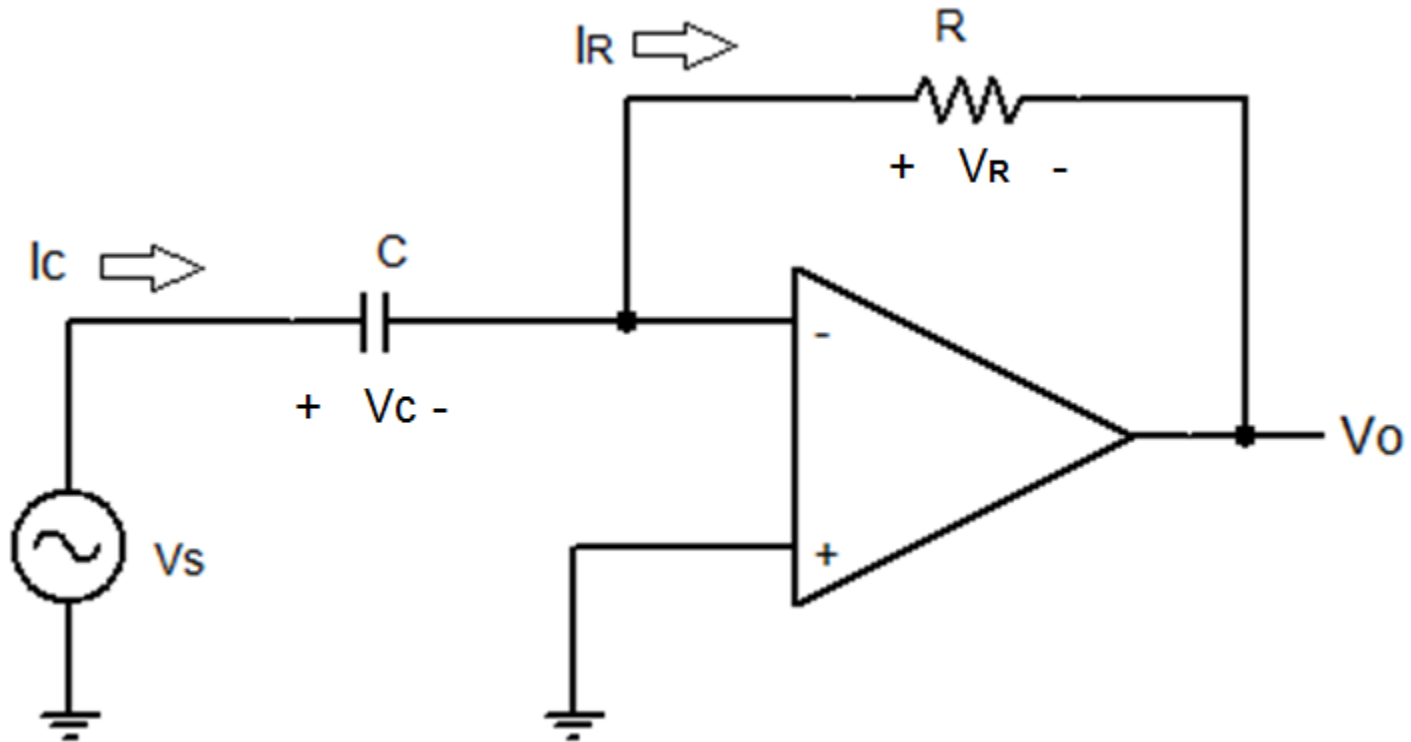


# Capacitors

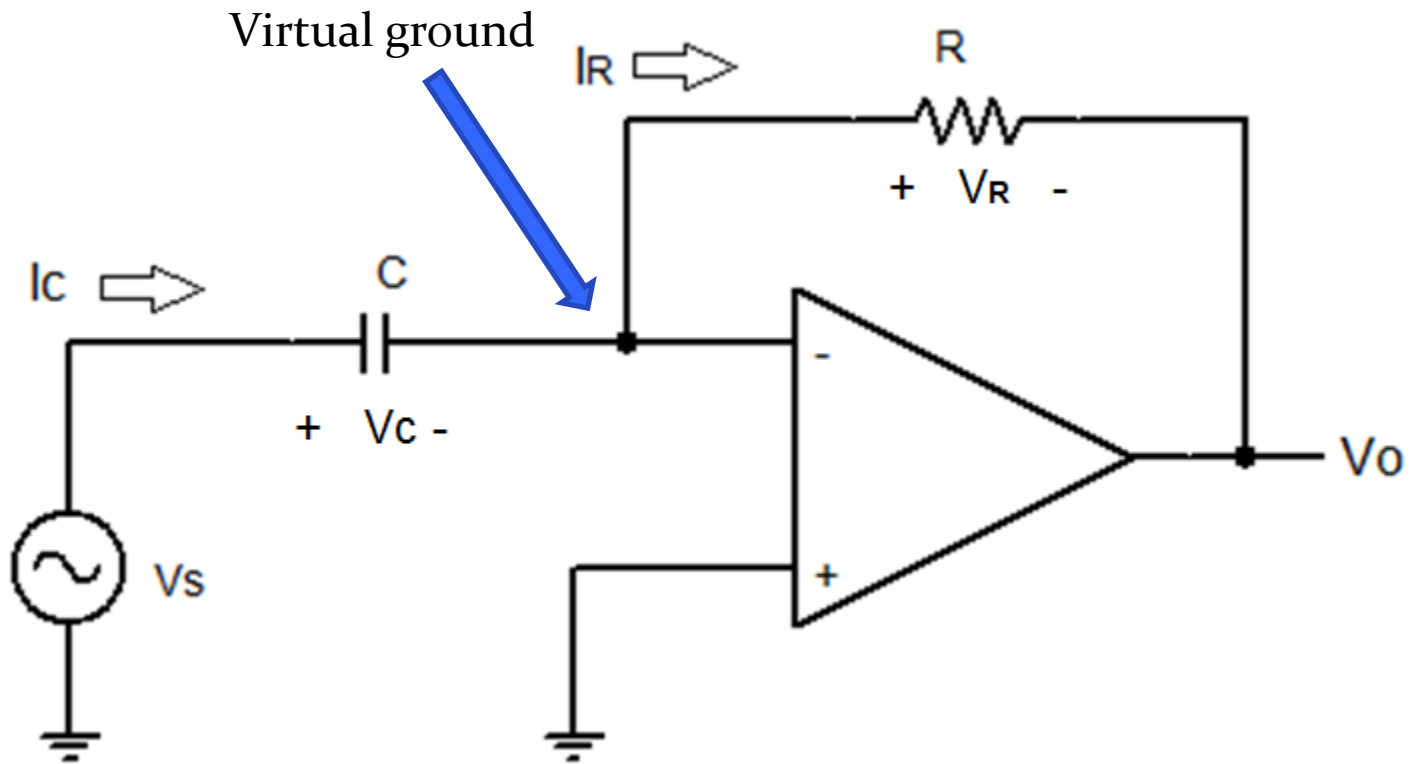
$$i_C(t) = C \frac{dv_C}{dt}$$

$$v_C(t) = \frac{1}{C} \int_{t_o}^{t_1} i_C(t) dt + v_C(t_o)$$

# Differentiator



# Ideal Op Amp Model



# Analysis

- Since current is not allowed to enter the input terminals of an ideal op amp.

$$i_C(t) = i_R(t)$$

$$v_C(t) = v_S(t)$$

$$i_C(t) = C \frac{dv_C}{dt} = C \frac{dv_S}{dt}$$

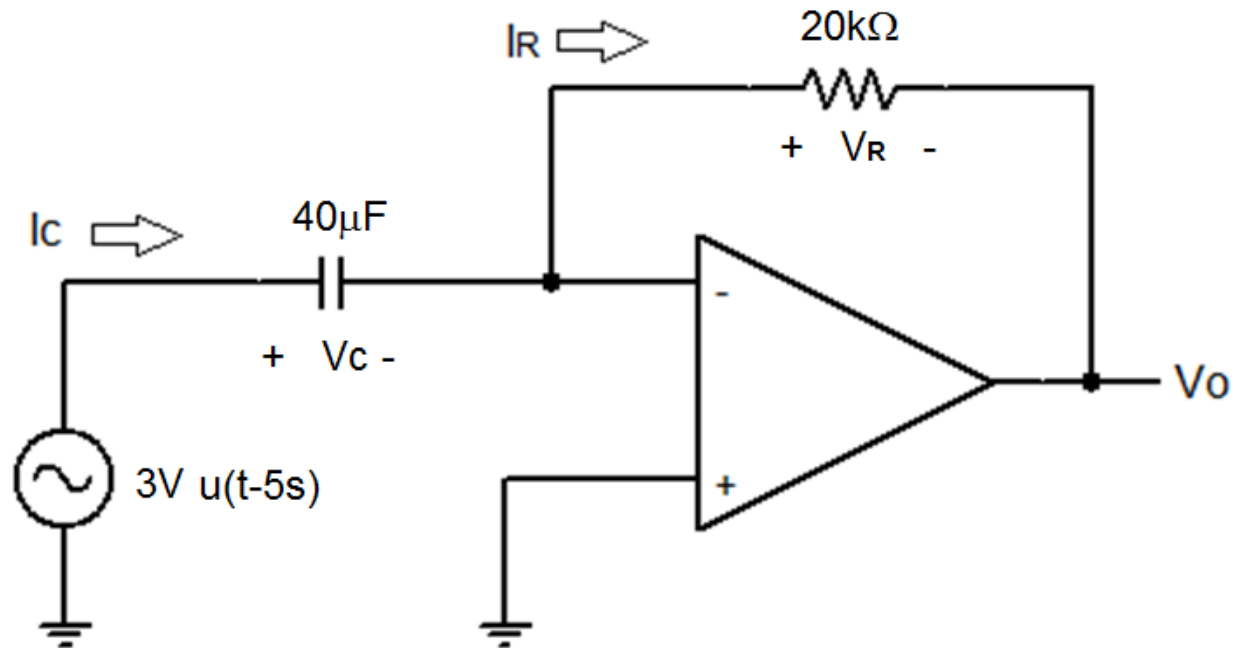
$$i_R(t) = -\frac{v_o}{R}$$

$$-\frac{v_o}{R} = C \frac{dv_S}{dt}$$

$$v_o(t) = -RC \frac{dv_S(t)}{dt}$$

# Example 01...

- Suppose  $v_S(t) = 3V u(t-5s)$ 
  - The voltage source changes from 0V to 3V at  $t = 5s$ .
    - Initial condition of  $V_C = 0V$  when  $t < 5s$ .
    - Final condition of  $V_C = 3V$  when  $t > 5RC$ .



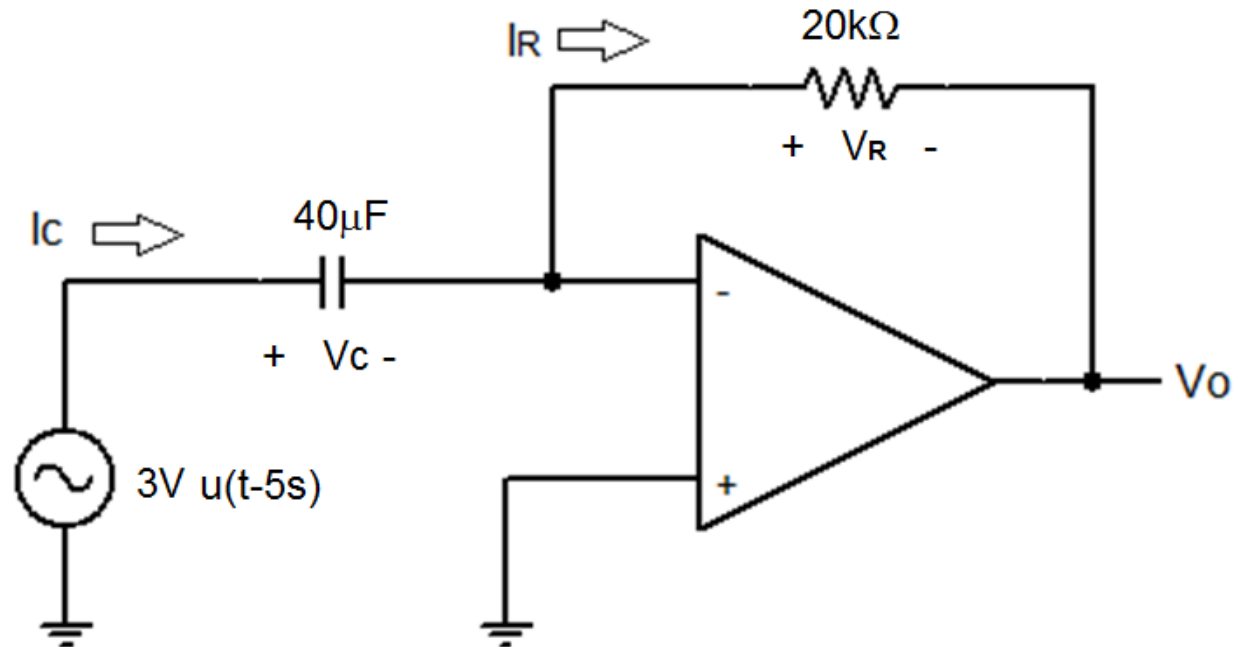
# ...Example 01...

$$v_C(t) = 0V \quad \text{when } t < t_0$$

$$v_C(t) = V_{C_{initial}} + (V_{C_{final}} - V_{C_{initial}}) e^{-(t-t_0)/\tau} \quad \text{when } t > t_0$$

$$v_C(t) = 0V + (3V - 0V) e^{-(t-5s)/0.8s} \quad \text{when } t > t_0$$

$$v_C(t) = 3V e^{-(t-5s)/0.8s} \quad \text{when } t > 5s$$



# ...Example 01

$$v_o(t) = -RC \frac{dv_C(t)}{dt}$$

$$v_o(t) = 0V \quad \text{when } t < 5s$$

$$v_o(t) = 0V \quad \text{when } t > t_o + 5\tau, \text{ where } \tau = RC$$

$$v_o(t) = 0V \quad \text{when } t > 5s + 5(20k\Omega)(40\mu F) = 9s$$

$$v_o(t) = \frac{-1}{0.8s} (-20 \times 10^3 \Omega)(40 \times 10^{-6} F)(3V) e^{-(t-5s)/0.8s}$$

$$v_o(t) = 3V e^{-(t-5s)/0.8s}$$

# Example 02

- Let  $R = 2 \text{ k}\Omega$ ,  $C = 0.1 \mu\text{F}$ , and  $v_s(t) = 2V \sin(500t)$  at  $t = 0\text{s}$

Since  $v_C(t) = v_s(t)$

$$v_o(t) = -RC \frac{dv_s}{dt}$$

$$v_o(t) = -(2000\Omega)(10^{-7} \text{ F}) \frac{d[2V \sin(500t)]}{dt}$$

$$v_o(t) = (-0.2\text{ms})(2V)(500)\cos(500t)$$

$$v_o(t) = -0.2V \cos(500t) \quad \text{when } t > 0\text{s}$$

$$v_o(t) = 0V \quad \text{when } t < 0\text{s}$$



# Cosine to Sine Conversion

$$v_o(t) = -0.2V \cos(500t)$$

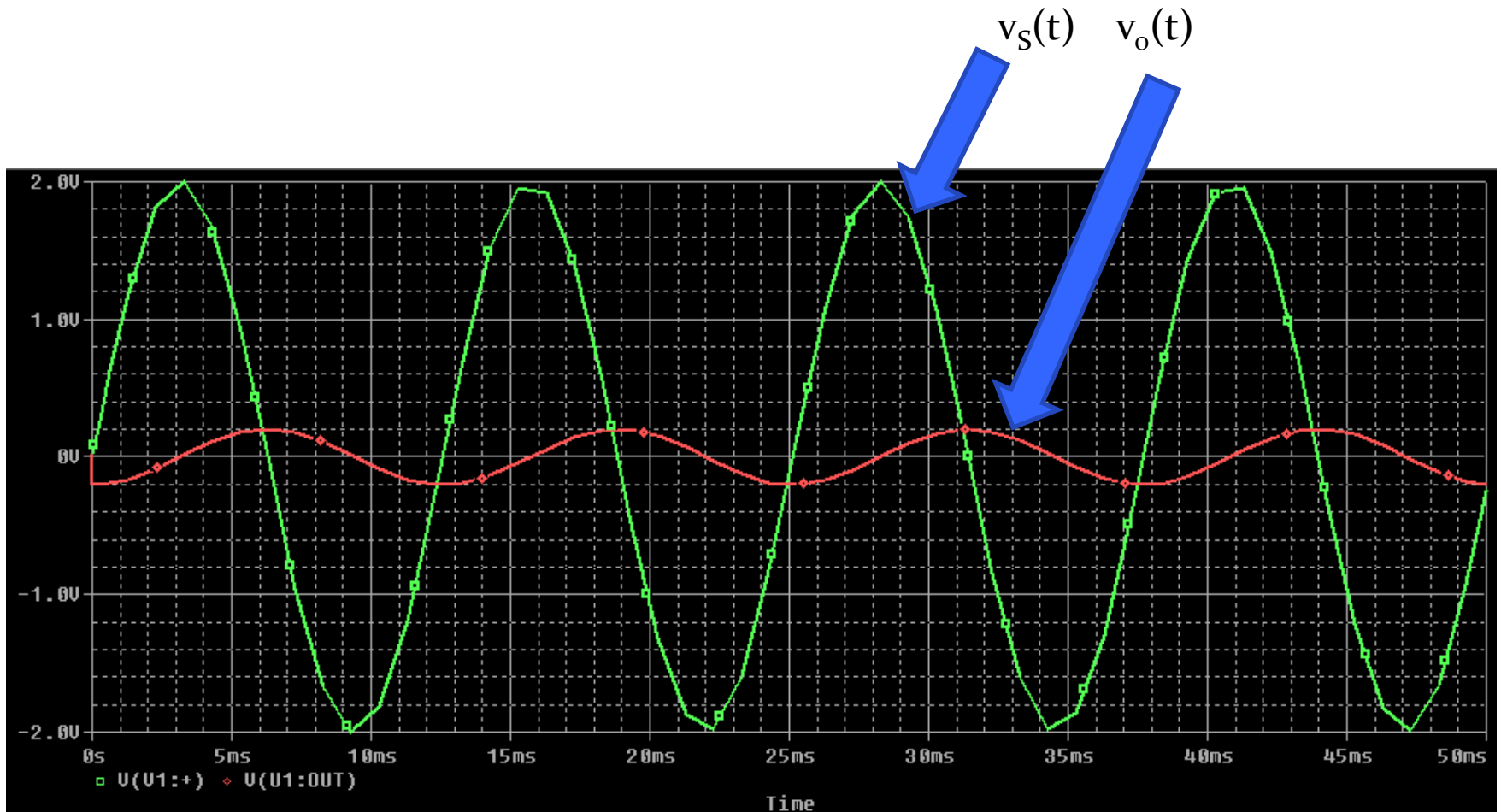
$$v_o(t) = -0.2V \sin(500t + 90^\circ)$$

$$v_o(t) = 0.2V \sin(500t + 90^\circ - 180^\circ)$$

$$v_o(t) = 0.2V \sin(500t - 90^\circ)$$

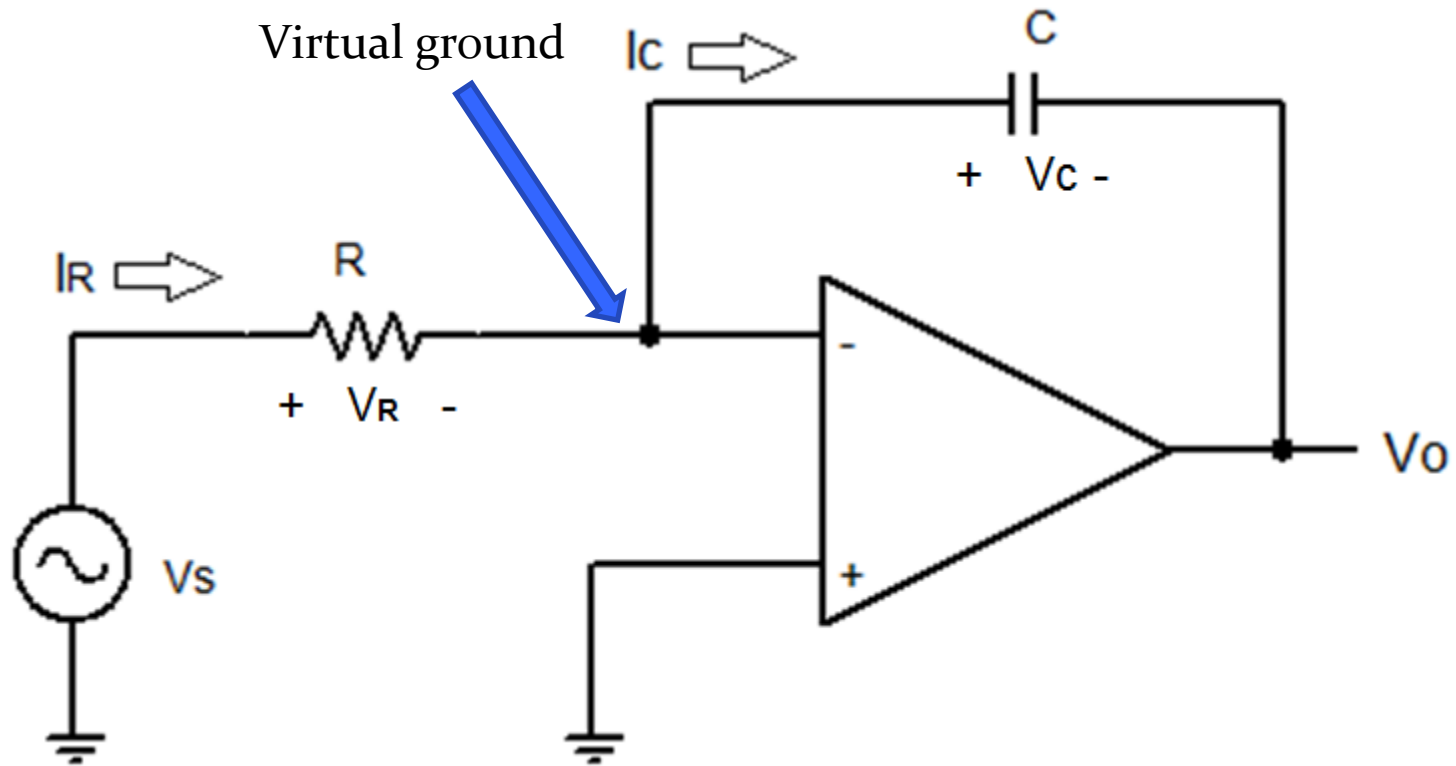
As  $v_s(t) = 2V \sin(500t)$  , the output voltage lags the input voltage by 90 degrees.

# PSpice Simulation

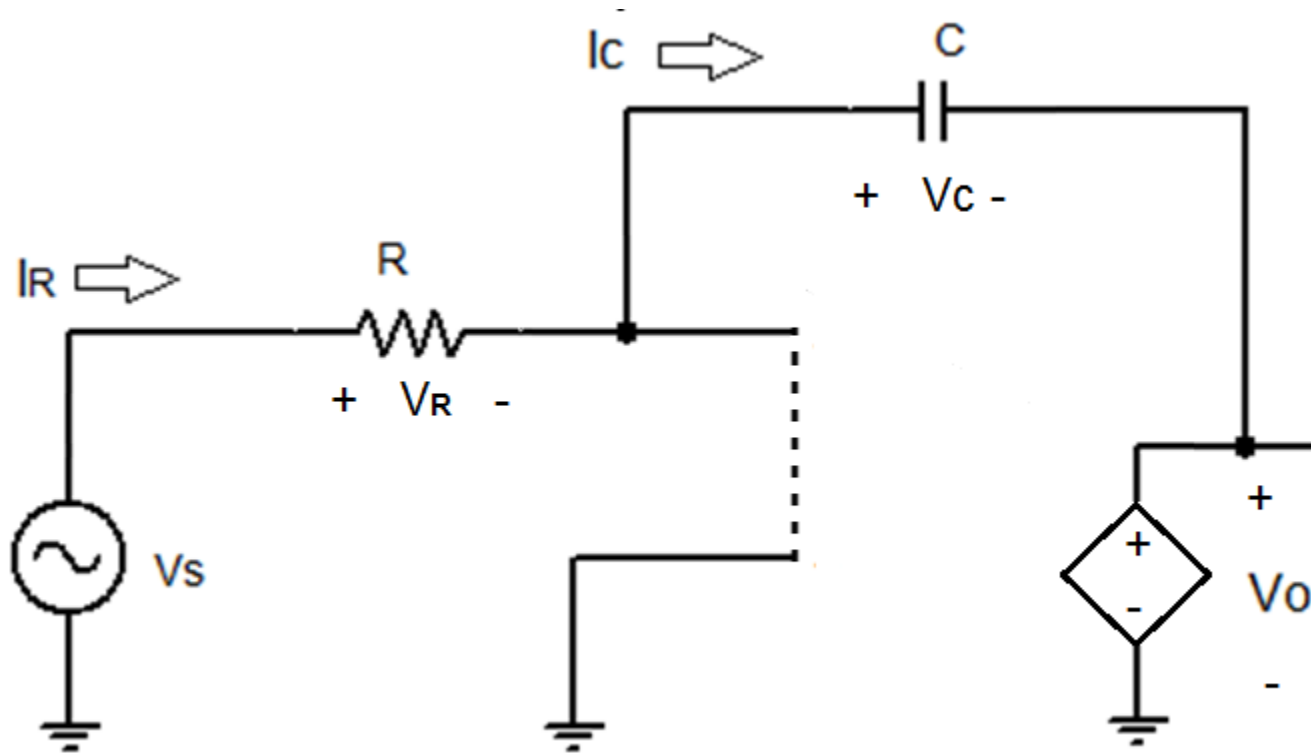


Shows the 90 degree phase shift as well as the attenuation.

# Integrator

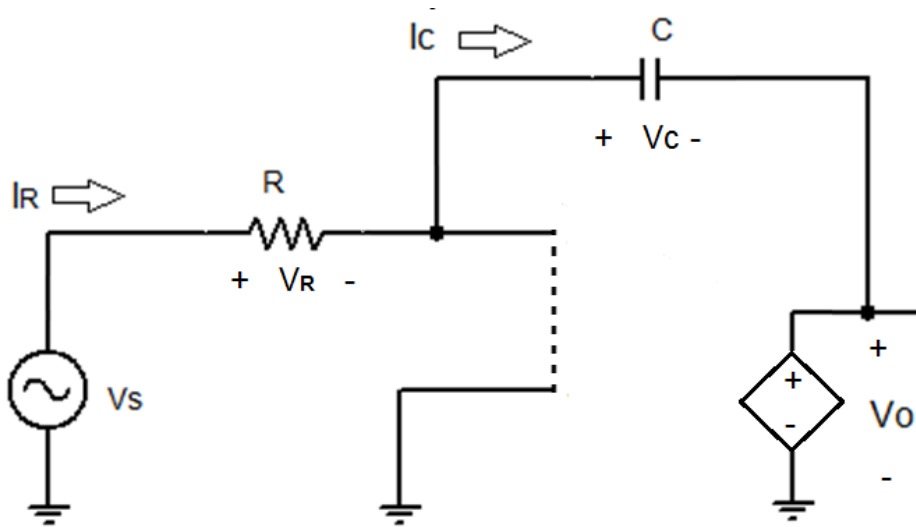


# Op Amp Model



# Integrator

- Op-Amp Model:



$$i_R = \frac{v_S(t) - v_1}{R} = \frac{v_S(t)}{R}$$

$$i_C = C \frac{dv_C}{dt}$$

$$v_C(t) = v_1 - v_o(t) = -v_o(t)$$

$$i_R - i_C = 0 \text{ mA}$$

$$\frac{v_S(t)}{R} - C \frac{d[-v_o(t)]}{dt} = 0$$

$$\frac{dv_o(t)}{dt} + \frac{v_S(t)}{RC} = 0$$

$$v_o(t_2) = \frac{-1}{RC} \int_{t_1}^{t_2} v_S(t) dt + v_o(t_1)$$

# Example 03

- Let  $R = 25 \text{ k}\Omega$ ,  $C = 5 \text{ nF}$ ,  $v_s(t) = 3V \sin\left(6.24k \frac{\text{rad}}{\text{s}} t\right)$  at  $t=0\text{s}$

$$V_o(t_2) = \frac{-1}{RC} \int_{t_1}^{t_2} V_{in}(t) dt + V_o(t_1)$$

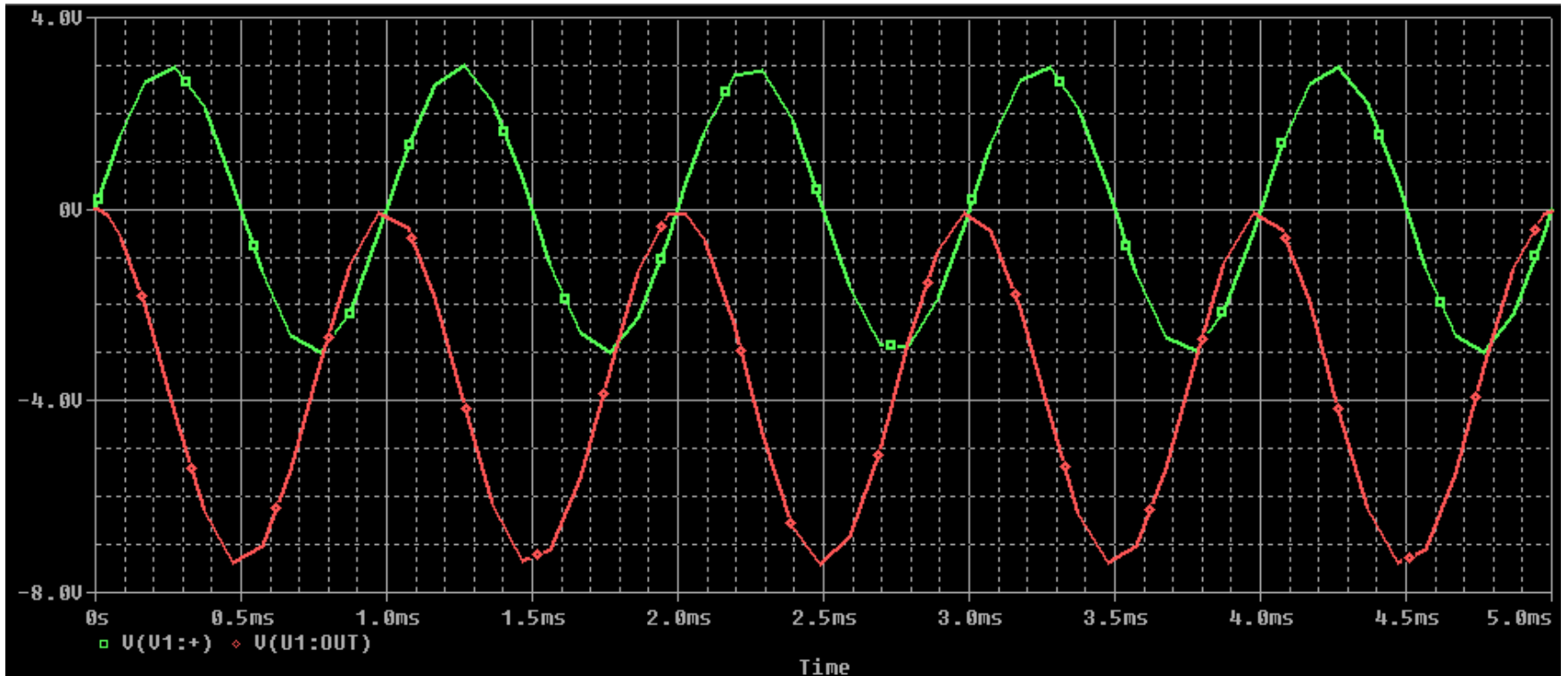
$$V_o(t_2) = \frac{-1}{25\text{k}\Omega(5\text{nF})} \int_{t_1}^{t_2} 3V \sin\left(6.24k \frac{\text{rad}}{\text{s}} t\right) dt$$

$$V_o(t_2) = 3.85V \cos\left(6.24k \frac{\text{rad}}{\text{s}} t\right) \Big|_{t_1}^{t_2} + V_o(t_1)$$

$$V_o(t_2) = 3.85V \sin\left(6.24k \frac{\text{rad}}{\text{s}} t_2 + 90^\circ\right) - 3.85V \text{ when } t_1 = 0\text{s}$$

since  $v_o(t) = -v_C(t)$  and the voltage across a capacitor can't be discontinuous.

# PSpice Simulation



Shows that the output voltage leads the input voltage by +90 degree and the voltage offset due to the  $V_o(t_1)$  term.

# Summary

- Differentiator and integrator circuits are 1<sup>st</sup> order op amp circuits.
  - When the C is connected to the input of the op amp, the circuit is a differentiator.
    - If the input voltage is a sinusoid, the output voltage lags the input voltage by 90 degrees.
      - The output voltage may be discontinuous.
  - When the C is connected between the input and output of the op amp, the circuit is an integrator.
    - If the input voltage is a sinusoid, the output voltage leads the input voltage by 90 degrees.
      - The output voltage must be continuous.