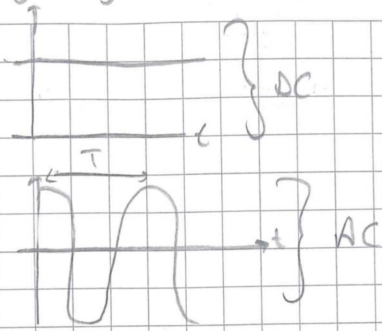
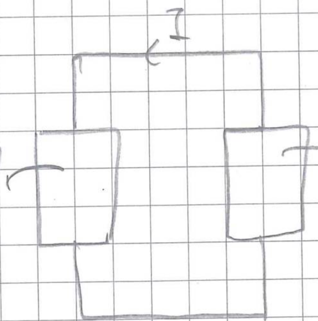
* BME 2301 - CIRCUIT THEORY *

(Dr. Görkem Serbes / gserbes@yildiz.edu.tr) Book: Engineering Circuit Analysis / W. Hayt





not fix

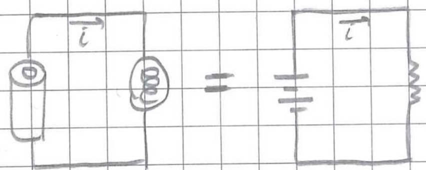
→ fix (controller port)

* Abstraction *

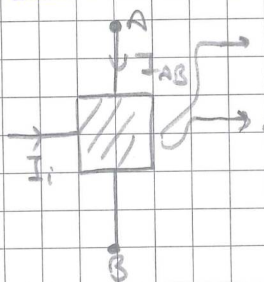
Lumped circuit → battery

analog abstraction

digital abstraction (not continous)



→ Transistors:



→ open : no current and there must be some voltage

→ short (closed) : wire , voltage is 0

* If we can storage energy, so we have memory. → like counters

* if we have digital abstraction, we need to logic and we don't have memory.

→ Circuit Types:

① Bread boards

② Printed Circuit boards (PCBs)

③ Integrated Circuit (ICs)

NOTE: from heart signals → ECG
muscles signals → EMG
brain signals → EEG
eyes signals → EOG
and these signals are continous signals; we filtered them and get rid of the noises. They are analog abstractions.

unwanted parts in some signal

→ **Linear vs Nonlinear :**

A linear system is predictable and easy to work with

Homogeneity

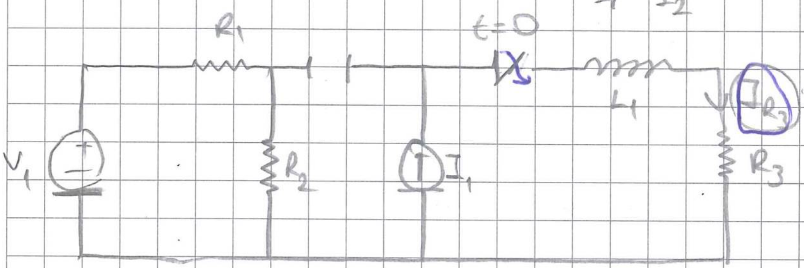
$2x\, V_{I_1} \longrightarrow \boxed{S} \longrightarrow V_{O_1}$

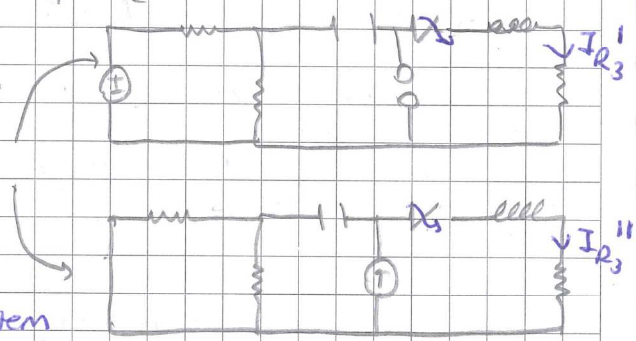
Additivity

$V_{I_1} \longrightarrow \boxed{S} \longrightarrow V_{O_1}$

$V_{I_2} \longrightarrow \boxed{S} \longrightarrow V_{O_2}$

$V_{I_1} + V_{I_2} \longrightarrow V_{O_1} + V_{O_2}$





$I_{R_3} = I_{R_3}' + I_{R_3}''$ } Linear System

* Analysis a system is already given system and solve for a part in it but designing a system, you have a problem and you need to think and design a solution.

→ **Electricity :** high ⟶ low

* Electricity flows from high voltage to low voltage
* Electricity flows in the opposite direction of electron flow.

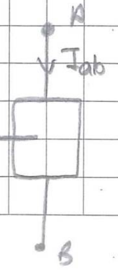
→ **Current** is the movement of charge.

$$i = dq/dt$$

$$q(t) = \int_{t=0}^{t} i(\tau)\, d\tau + q(t_0)$$

→ **Voltage** is the energy required to move a unit charge through a element measured in volts (V)

$$V_{ab} = -V_{ba}$$



→ If the energy high at A, it flows.

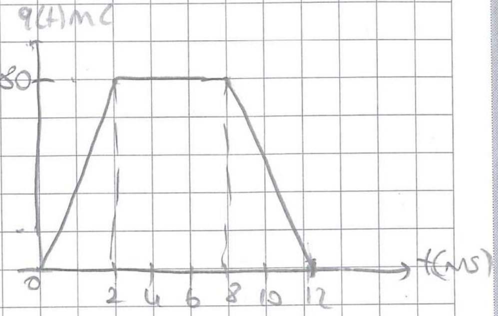
This element voltage-current element

* Define a current → we need to know magnitude and direction.

* Define a voltage → we need to know magnitude and polarity.

Ex: @ $t = 1ms$ → positive movement

$i(t) = \frac{dq(t)}{dt} = \frac{\Delta q}{\Delta t} = \frac{80mC - 0}{2mS - 0} = \frac{80mC}{2mS} = \underline{\underline{40A}}$



@ $t = 6mS$

there is no charge, $\underline{\underline{A = 0}}$

@ $t = 10mS$ → negative movement

$i(t) = \frac{dq(t)}{dt} = \frac{\Delta q}{\Delta t} = \frac{0 - 80mC}{12mS - 8mS} = \frac{-80mC}{4mS} = \underline{\underline{-20A}}$

→ Power is the rate of energy change in the circuit. ⇒ $P = V \cdot i$

⊛ Passive Sign Convention:

① Positive: If the current enters from (+) side then the power absorbes energy.

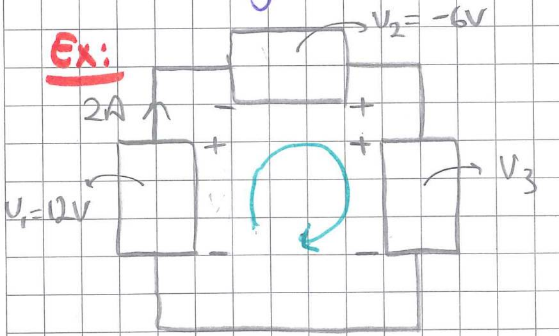
$P = (+)(V)(i)$

② Negative: If the current enters from (−) side then the power supplies energy.

$P = (-)(V)(i)$

* Tellegen's Theorem → $\Sigma P = 0$ → because of energy conservation.

Ex:



$V_3 = ?$

$P_1 = (-)(12V)(2A) = -24W$

$P_2 = (-)(-6V)(2A) = 12W$

$P_3 = (+)(V_3)(2A) = 2V_3$

by Tellegen's $\Sigma P = 0$

$-24 + 12 + 2V_3 = 0$

$2V_3 = 12 \Rightarrow \underline{\underline{V_3 = 6V}}$

NOTE: $P(t) \underset{\text{integrate}}{\overset{\text{derivate}}{\rightleftarrows}} W(t)$

→ Circuit Componens :

1. Active elements → independent power sources
   ± voltage ↑ current
   → dependent power sources
   ◇± → voltage ◇↑ → current

2. Passive elements → resistors
   → capacitors
   → inductors

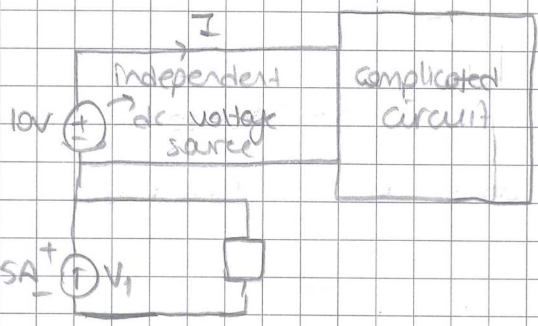
3. Measurement devices → ampermeters → voltmeters

4. Ground (reference point)

5. electric wire

6. switches

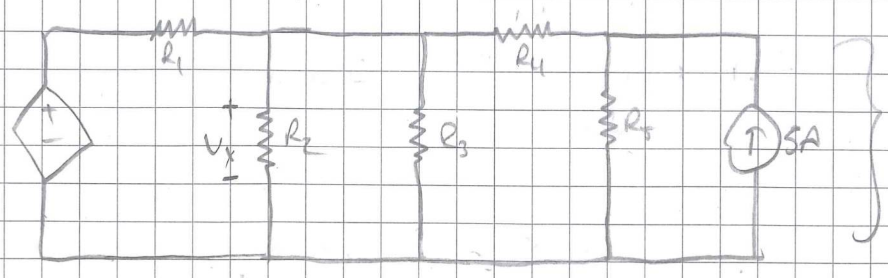
7. protective devices → fuse



* Types of dependent power sources *

voltage controlled → voltage source (VCVS)
→ current source (VCCS)

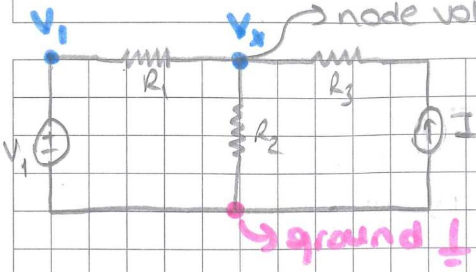
current controlled → voltage source (CCVS)
→ current source (CCCS)



Voltage controlled voltage source circuit

* in control circuits we must use switches.

in AC { * time differences between in two wave → phase difference ($\varnothing$)



before solved, choose a reference point.

! The volt value at the two ends of the non-current resistor is the same. !

if we have two resistances (one of them is too big than the other) which are connect to each other parallels, their equivalent resistance value approximates to small one.

Don't forget to choose reference point in the circuits. (in real life)

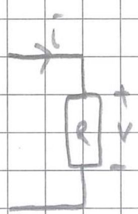
→ Resistivity depends on Mobility and points width (ρ)

$R = \underline{\underline{\rho}} \frac{L}{A}$ $V = I \cdot R$

→ Conductance (G) inverse of the resistivity. (conductivity (σ))

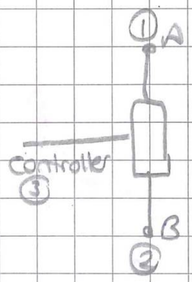
$G = R^{-1} = \frac{i}{V}$ $G = \frac{A\sigma}{L}$

★ Short Circuit ★



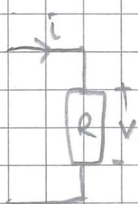
if the resistor is a perfect conductor then,

$R_{in} = 0\Omega \rightarrow V = 0$ volts



$V_{ab} = V_A - V_B$
$= 0$
short circuit

★ Open Circuit ★

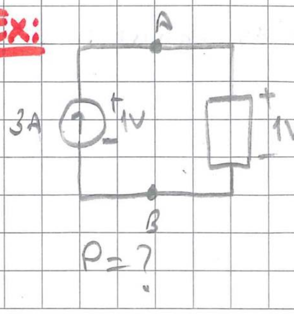


if the resistor is a perfect insulator then,

$R_{in} = \infty\Omega \rightarrow i = \lim_{R \to \infty} \frac{V}{R} = 0$ * we may have voltage

* there is no power dissipated in open & short circuits.

Ex:



$P = ?$

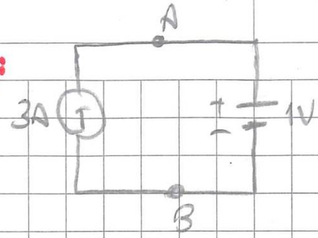
$V_{AB} = 1V$

$V_{CS} = V_{AB} = 1V$ } by the KVL
across the current source

$P = (-)(1V)(3A)$

$= \underline{\underline{-3 \text{ watts}}}$ supplied power

Ex:

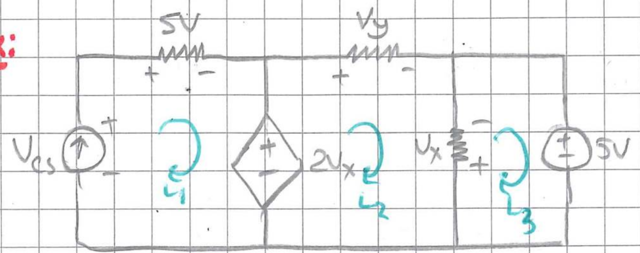


$P_{cs} = (-)(3A)(1V) = -3\,watt \rightarrow$ supplied power

$P_{VS} = (+)(3A)(1V) = 3\,watt \rightarrow$ absorbed power

↳ in this source, battery charged by current source.

Ex:



$V_{cs}, V_x, V_y = ?$

By the KVL / Tellegen's Theorem

for $L_3 \Rightarrow +V_x + 5V = 0$

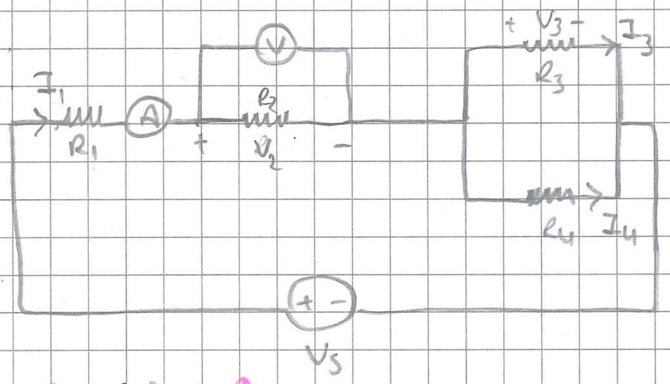
$\underline{V_x = -5V}$

for $L_2 \Rightarrow -2V_x + V_y - V_x = 0$

$\underline{V_y = -15V}$

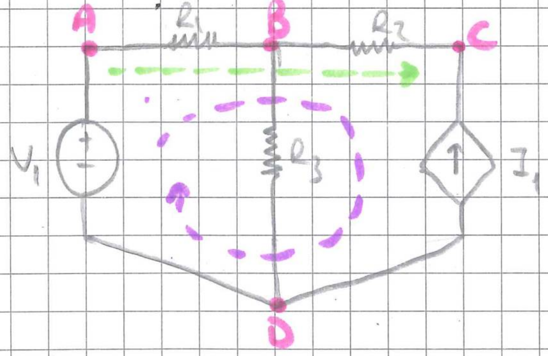
for $L_1 \Rightarrow -V_{cs} + 5V + 10V = 0$ (2x)

$\underline{V_{cs} = -5V}$



ampermeter $\rightarrow R_{in} = 0$

voltmeter $\rightarrow R_{in} = \infty$

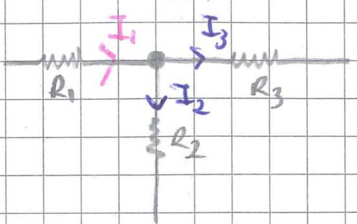


A, B, C, D $\rightarrow$ nodes

ABC $\rightarrow$ path

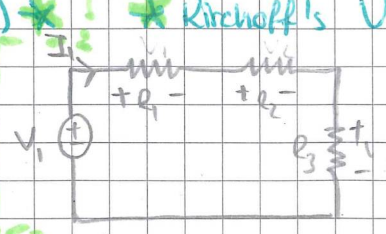
ABCDA $\rightarrow$ loop

★ Kirchoff's Current Law (KCL) ★



$\sum_{node} i_{enter} = \sum_{node} i_{leave} \Rightarrow \boxed{\sum_{n=1}^{N} i_n = 0}$

★ Kirchoff's Voltage Law (KVL) ★

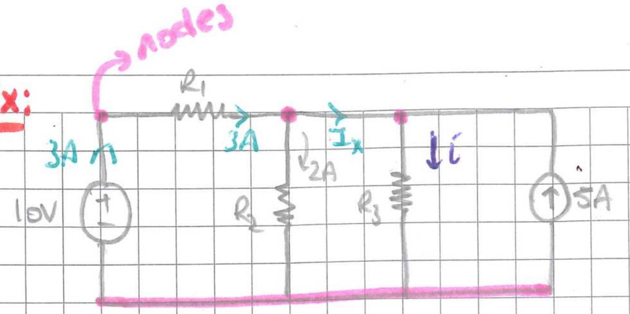


$-V_1 = V_{R_1} + V_{R_2} + V_{R_3}$

$V_1 + V_{R_1} + V_{R_2} + V_{R_3} = 0$

$\sum V_{drops} = \sum V_{rises} \Rightarrow \boxed{\sum_{n=1}^{N} V = 0}$

Ex:



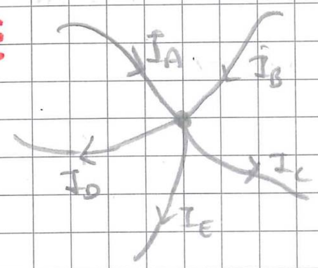
Voltage source supplies a current of 3A so,

$i = ?$

By the KCL $\Rightarrow$ $3A = I_x + 2A$ $\qquad$ $I_x + 5A = i$

$I_x = 1A$ $\qquad$ $\underline{i = 6A}$

Ex:

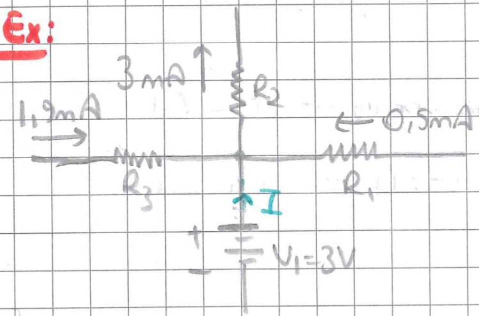


$I_A = 1A$, $I_C = 3A$, $I_D = 2A$, $I_E = 4A$ $\rightarrow$ $I_B = ?$

$I_A + I_B = I_D + I_E + I_C$

$I_B = I_D + I_E + I_C - I_A$ $\Rightarrow$ $\underline{I_B = 4A}$

Ex:



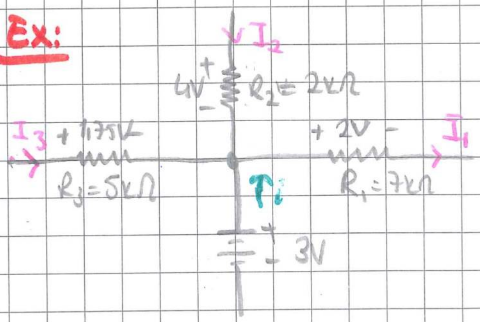
$I = ?$, $P_I = ?$

$I + 1{,}9mA + 0{,}5mA = 3mA$ $\Leftarrow$ KCL

$\underline{I = 0{,}6mA}$

$P_I = (-)(0{,}6mA)(3V) = \underline{-1{,}8mW}$ supplying

Ex:

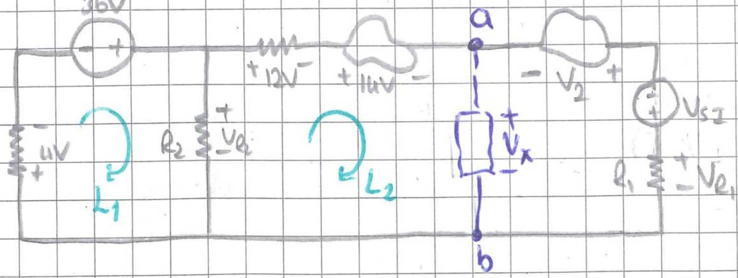


$i = ?$

$I_3 + I_2 - I_1 + i = 0$

$-\left(\frac{1{,}75V}{5k\Omega} + \frac{4V}{2k\Omega} - \frac{2V}{7k\Omega}\right) = i$ $\qquad$ $\underline{i = -2{,}0643mA}$

Ex:



$V_{R_2} = ?$ $\qquad$ $V_{ab} = ?$

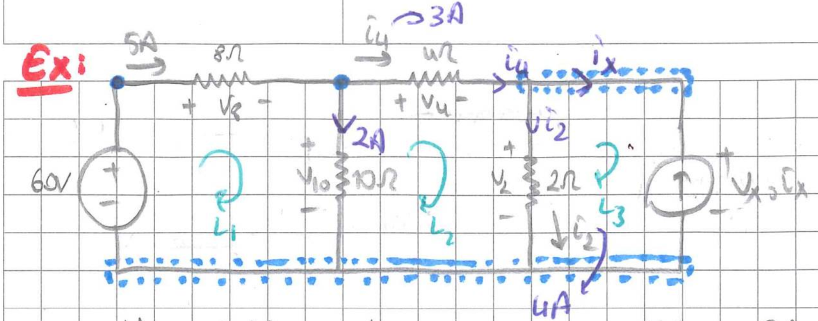
KVL L_1 $\Rightarrow$ $4V - 36V + V_{R_2} = 0$

$\underline{V_{R_2} = 32V}$

$-32V + 12V + 14V + V_x = 0$ $\Leftarrow$ KVL L_2

$\underline{V_x = 6V}$

Ex:



$V_x = ?$

$V_8 = 5A \cdot 8\Omega = 40V$

$KVL \to L_1 \Rightarrow -60 + 40 + V_{10} = 0$

$V_{10} = 20V$

$V_{10} = i_{10} \cdot 10\Omega \Rightarrow i_{10} = 2A$

$KCL \Rightarrow 5A = i_4 + i_{10}$

$i_4 = 3A$

$V_4 = 3A \cdot 4\Omega$
$= 12V$

$KVL \to L_3 \Rightarrow -8V + V_x = 0$

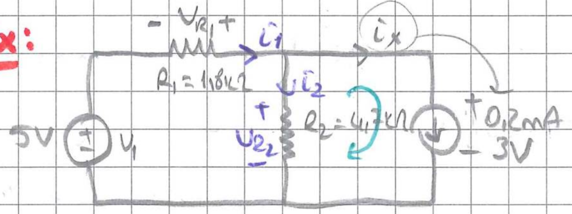
$\boxed{V_x = 8V}$

$KVL \to L_2 \Rightarrow -V_{10} + V_4 + V_2 = 0$

$-20 + 12 + V_2 = 0$
$V_2 = 8V$

$KCL \Rightarrow i_4 - i_2 + i_x = 0$
$i_x = 1A$

Ex:



$V_{R_1} = ?$

$L \Rightarrow -V_{R_2} + 3V = 0$

$V_{R_2} = 3V \Rightarrow i_2 = \frac{3V}{4{,}7k\Omega} = 0{,}638 mA$

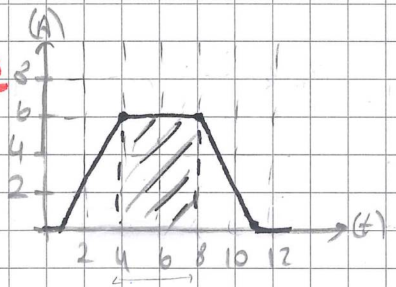
$i_1 = i_x + i_2$

$i_1 = 0{,}2mA + 0{,}638mA = 0{,}84 mA$

$V_{R_1} = (-)(0{,}84mA)(1{,}8k\Omega)$

$\boxed{V_{R_1} = (-) 1{,}5V}$ comes from, current enters (-) side

Ex:

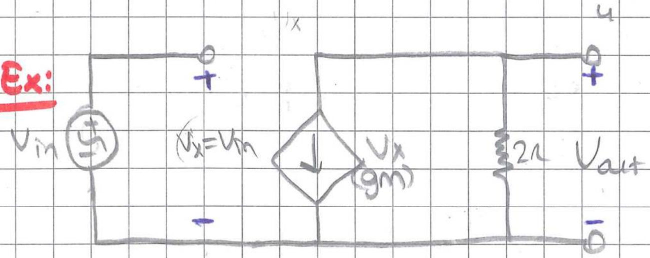


What is the net charge that passes through the element between t=4 and t=8?

$q = \int i(t)dt \rightarrow$ we should look the area

$= \int_4^8 i(t)dt = 4sec \cdot 6A = \underline{\underline{24C}}$

Ex:



$V_{in} = 3 \sin(\omega t)\, mV$
$gm = 10\, A/V$
$\Big\}\ V_{out} = ?$

angular frequency

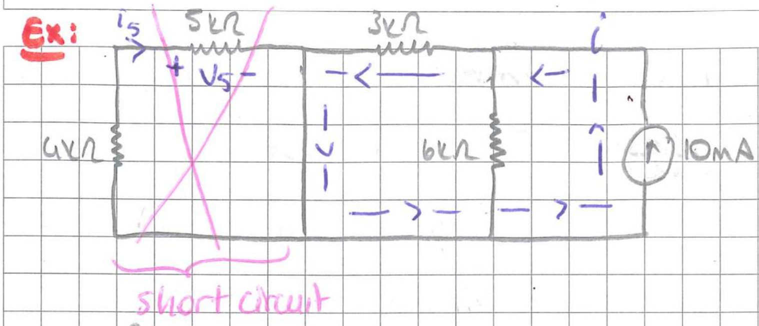
$V_{out} = (-)(2\Omega)(gm)(V_x) = (-)(2\Omega)(10A/V)(3 \sin(\omega t)\, mV)$

enters from negative side

$\underline{V_{out} = -60 \sin(\omega t)\, mV}$

* open circuit here, between + - $R_x = \infty$

Ex:

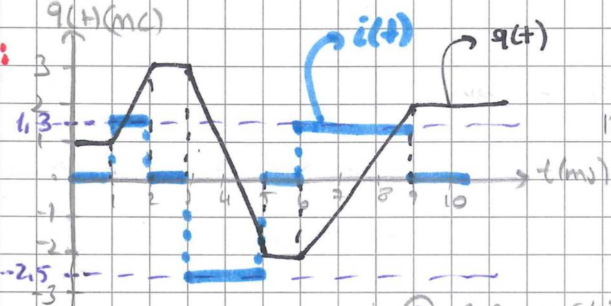


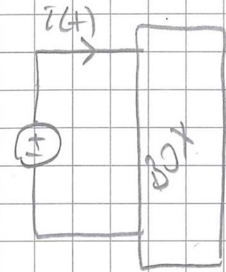
Determine the power absorbed by the 5kΩ resistor.

$P_{5k\Omega} = (-5)(V_5)(i_5) = \underline{0 \text{ watts}}$

because the short circuit

Ex:





The charge that enters the Box is shown. Calculate and sketch the current flowing by the box between 0-10ms.

$q(t) = \frac{dq(t)}{dt}$

@ 0-1 → $i(t)=0$

@ 1-2 → $i(t) = \frac{\Delta q}{\Delta t} = 2A$

@ 2-3 → $i(t)=0$

@ 3-5 → $i(t) = \frac{(-2)-3}{2} = -2,5A$

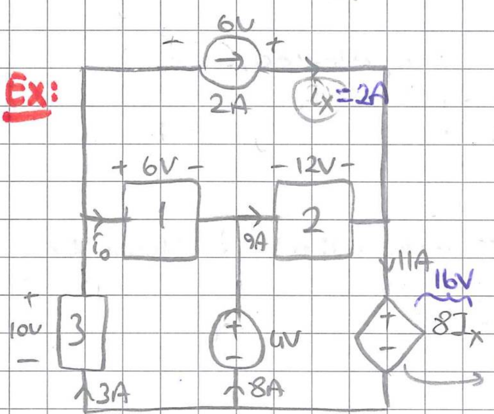
@ 5-6 → $i(t)=0$

@ 6-9 → $i(t) = \frac{2-(-2)}{3} = 1,33A$

@ 9-10 → $i(t)=0$

* ▬ is the sketch of solution

Ex:



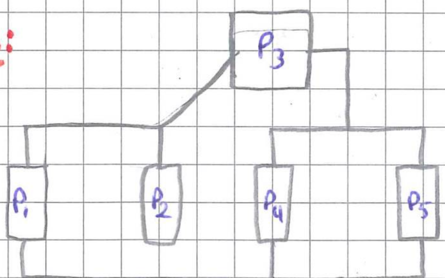
find the i_0 by the Tellegen's Theorem.

$P_1 = (6V)(i_0) = 6i_0 W$
$P_2 = (-)(12V)(9A) = -108W$
$P_3 = (-)(10V)(3A) = -30W$
$P_{4V} = (-)(4V)(8A) = -32W$
$P_{2A} = (-)(6V)(2A) = -12W$
$P_{dep} = (+)(16V)(11A) = 176W$

The total sum must be zero by Tellegen so,

$6i_0 = 6 \Rightarrow \underline{i_0 = 1A}$

Ex:



$P_1 = 50W$
$P_2 = 40W$
$P_3 = -10W$
$P_4 = 5W$

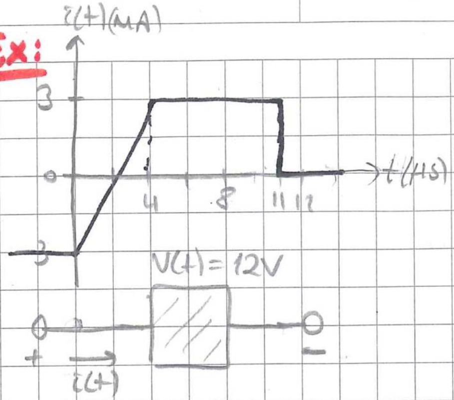
$P_5 = ?$

Tellegen's

$50W + 40W - 10W + 5W + P_5 = 0$

$\boxed{P_5 = 15W}$

Ex:



under the area

a) Compute the net charge between $t=2\mu s$ and $t=12\mu s$

$$i(t)=\frac{dq(t)}{dt}$$

$$q=\int_2^{12} i(t)\,dt=\frac{(2\mu s)(3mA)}{2}+(7\mu s)(3mA)$$

$$=\underline{24\,nC}$$

b) When does the element absorbed & supplied electrical energy?

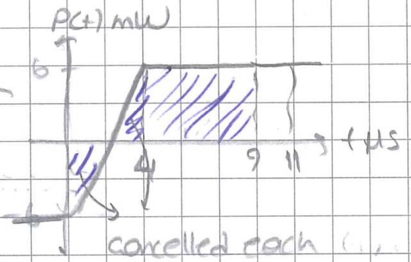
$P(t)=i(t)\cdot v(t) \rightarrow t@\ 0-2\mu s \Rightarrow P=(-) \rightarrow$ supplied

$@\ 2-11\mu s \Rightarrow P=(+) \rightarrow$ absorbed

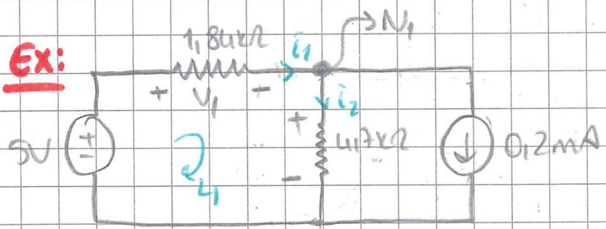
c) Compute the net energy absorbed by the element between $t=0\mu s$ and $t=9\mu s$

$$W=\int P(t)dt=\int_0^9 P(t)dt$$

$$=(5\mu s)(6mW)=\underline{30\,\mu Joule}$$



Ex:



$V_1=?$

KVL @ $L_1 \Rightarrow -5V+(1,84)(i_1)+(4,7)(i_2)=0$

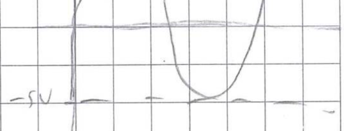
KCL @ $N_1 \Rightarrow i_1-i_2-0,2mA=0$

$1800\,i_1+4700\,i_2=5V$

$i_1-i_2=0,2mA$

$$\underbrace{\begin{bmatrix}1800 & 4700\\ 1 & -1\end{bmatrix}}_{a}\underbrace{\begin{bmatrix}i_1\\ i_2\end{bmatrix}}_{b}=\underbrace{\begin{bmatrix}5\\ 0,2\times10^{-3}\end{bmatrix}}_{c}$$

we solved these matrices on matlab



$$(a^{-1}*a)*b=a^{-1}*c$$

$$b=a^{-1}*c$$

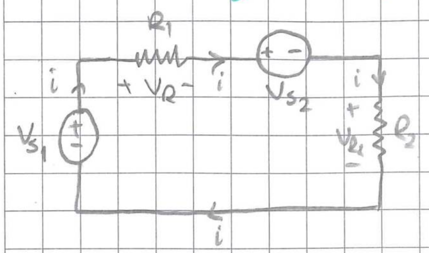
$i_1=0,918mA$

$i_2=0,7138mA$

$V_1=(+)(0,9138mA)(1,84k\Omega)$

$\underline{V_1=+1,65V}$

* The Single-Loop Circuit *



$-V_{s_1} + V_{R_1} + V_{s_2} + V_{R_2} = 0$

$V_{R_1} = R_1 \cdot i$ and $V_{R_2} = R_2 \cdot i$

$-V_{s_1} + R_1 i + V_{s_2} + R_2 i = 0$

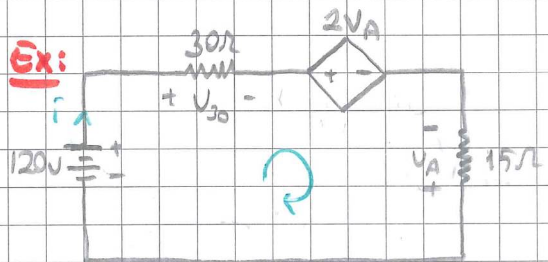
$$i = \frac{V_{s_1} - V_{s_2}}{R_1 + R_2}$$

① Analysis is the assumption of reference directions for unknown currents.

② Analysis is a choice of the voltage reference for each of the resistors.

③ Application of KVL to the only closed path.

Ex:



↳ voltage controlled voltage single-loop circuit

Compute the power absorbed in each element at the circuit.

1- KVL ⇒ $-120V + V_{30} + 2V_A - V_A = 0$

$V_{30} + V_A = 120V$

2- $V_{30} = (i) \cdot 30\Omega$

$V_A = -(i) \cdot 15\Omega$

$30i - 15i = 120V$

$\underline{i = 8A}$

3- $P_{120V} = (-)(8)(120) = -960W$

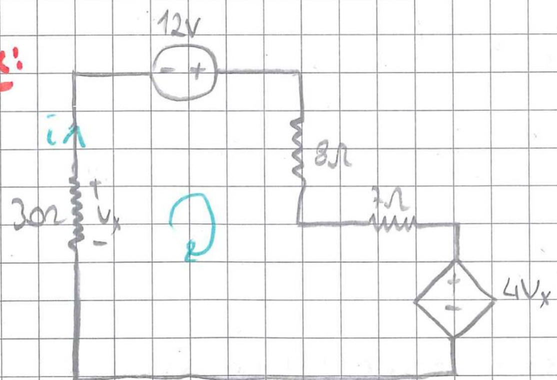
$P_{30\Omega} = i^2R = (64)(30) = 1920W$

$P_{2V_A} = (+)(8)(-240) = -1920W$

$P_{V_A} = i^2R = (64)(15) = 960W$

$\underline{P_T = 0W}$

Ex:



$V_x = (-)(30\Omega) \cdot i$

$i = -V_x/30$

$P_T = ?$

$-V_x - 12V + (8+7)i + 4V_x = 0$

$-V_x - \frac{15V_x}{30} + 4V_x = 12$

$V_x = \frac{24}{5}$ Volts

↳ $i = \frac{-4}{25}$ A

$P_{30\Omega} = 760mW$

$P_{12V} = 1{,}92W$

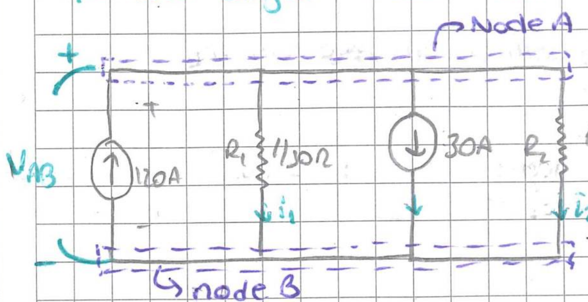
$P_{8\Omega} = 204{,}8mW$

$P_{7\Omega} = 179{,}2mW$

$P_{4V_x} = -3{,}072W$

$\underline{P_T = 0W}$

* The Single-Node-Pair Circuit *



Find the voltage, current and power associated with each element.

KCL @ node A → $120 - i_1 - i_2 - 30 = 0$

$i_1 + i_2 = 90A$

$30V_{AB} + 15V_{AB} = 90$

$V_{AB} = 2V$

$i_1 = \frac{V_{AB}}{1/30} = 60A$

$i_2 = \frac{V_{AB}}{1/15} = 30A$

↳ current sources like $120 - 30 = 90A$

$P_{120A} = (-)(2)(120A) = -240W$

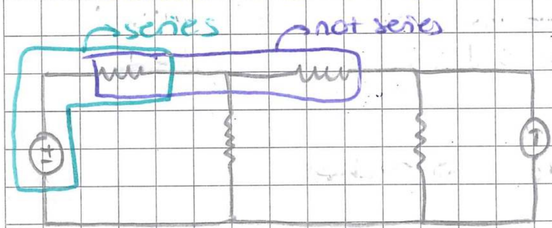
$P_{30A} = (+)(2V)(30A) = 60W$

$P_{R_1} = 60^2 \cdot 1/30 = 120W$

$P_{R_2} = 30^2 \cdot 1/15 = 60W$

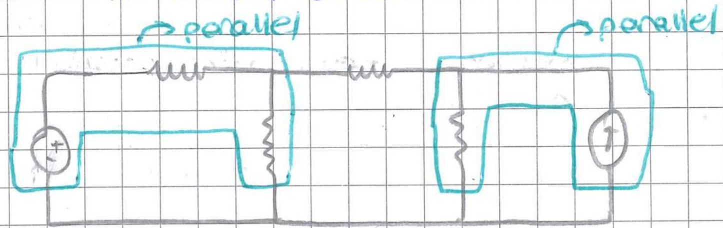
$P_T = \underline{\underline{0W}}$

* Series Circuits *



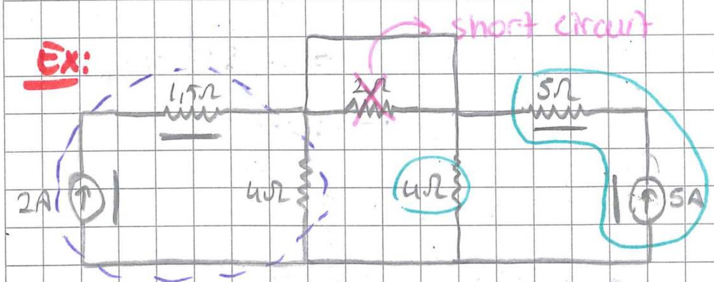
→ all elements in a circuit (loop) that carry the same current.

* Parallel Circuits *



→ all elements in a circuit (loop) that have a common voltage across them.

Ex:



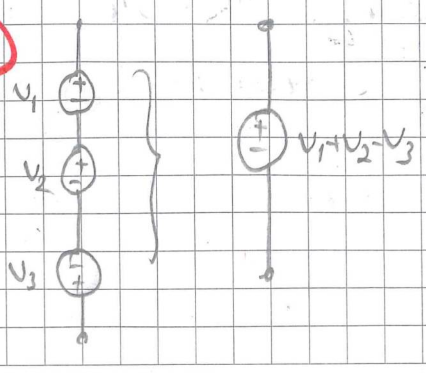
write the parallels & series elements.

2A & 1,5Ω
5A & 5Ω } individual series

5A & 5Ω parallel with 4Ω
2A & 1,5Ω parallel with 4Ω

groups.

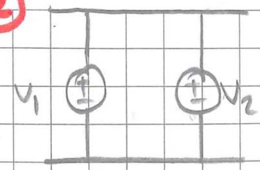
NOTE: 1



→ We can replace voltage source in series with a single equivalent source.

$\Sigma V_n = V_{equivalent}$

2

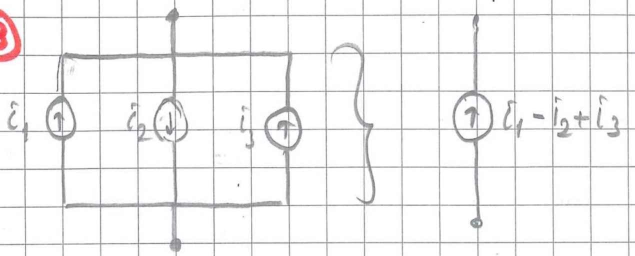


→ unless $V_1 = V_2 = \dots$ this circuit is not valid for ideal sources

→ All real voltage source have internal resistance and are usually not exactly equal.

→ Current will flow from the higher source to the lower source until equilibrium is reached.

3



→ We can replace current sources in parallel with a single equivalent source.

$$\sum i_n = i_{\text{parallel equivalent}}$$

4 → in series resistances are added $\sum R_n = R_1 + R_2 + \dots + R_n = R_{\text{series equivalent}}$

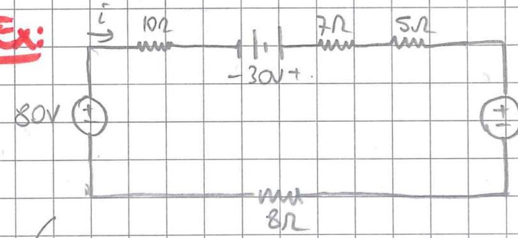
→ in parallel resistance $\sum 1/R_n = 1/R_1 + 1/R_2 + \dots + 1/R_n = R_{\text{parallel equivalent}}$

→ $R_1 \| R_2 \| R_3$ parallel symbol

5 → if G is used instead of R ; in series → the reciprocal of the equivalent conductance is equal to the sum of the reciprocal of each of the conductors in series.

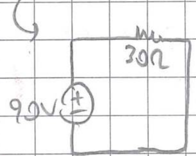
$$1/G_{eq} = 1/G_1 + 1/G_2$$ → inverse R G → mho

Ex:



$i = ?$, $P_{80V} = ?$

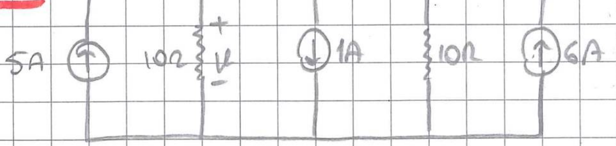
80V, -30V+, 20V } 90V = V_{TOTAL}



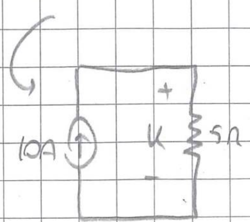
$R_{eq} = 10 + 7 + 5 + 8 = 30\Omega$ → $i = \frac{90V}{30} = \underline{3A}$

$P_{80V} = -(80)(3A) = \underline{-240W \text{ delivered}}$

Ex:



Determine V in this circuit by first combining the three current sources, then the two 10 ohm resistors.



$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{10} \Rightarrow R_{eq} = 5\Omega$

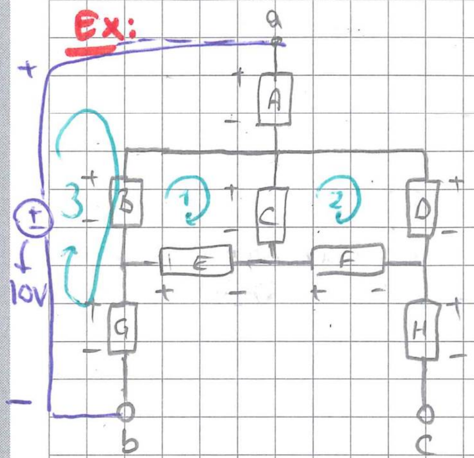
$I_{Total} = 5 - 1 + 6 = 10A$

$V_T = (10A)(5\Omega) = \underline{50V}$

As you increase the number of resistors in series; Req increase.

As you increase the number of resistors in parallel; Req decrease.

Ex:



For the diagram above the following voltages are specified;

$V_A = -2V,\ V_D = 2V,\ V_E = 3V,\ V_G = 4V,\ V_H = 2V,\ V_{ab} = 10V$

Find the value of V_C in volts.

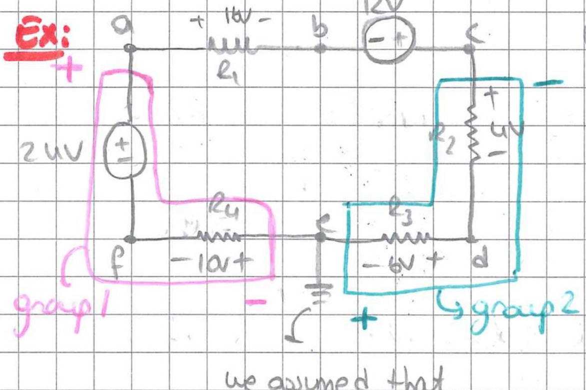
KVL @ loop 1 $\rightarrow -V_B + V_C - V_E = 0 \rightarrow V_C = V_B + V_E$

@ loop 2 $\rightarrow -V_C + V_D - V_F = 0 \rightarrow V_C = V_D - V_F$

@ loop 3 $\rightarrow -10V + V_A + V_B + V_G = 0 \rightarrow V_B = 8V$

$V_C = V_B + V_E \Rightarrow V_C = 8V + 3V = \underline{11V}$

Ex:



Write loop equations, find V_{ae}, V_{ec}.

①. way

1) $V_{ae} = V_a - V_e$ $\qquad V_e = 0 \rightarrow$ for the reference point

$V_a = V_f + 24V = 14V$
$\quad (-10V)$

$V_{ae} = V_a - V_e = 14V - 0 \rightarrow \underline{V_{ae} = 14V}$

2) $V_{ec} = V_e - V_c$

$V_b = V_a - 16V = -2V$

$V_c = V_b + 12V = 10V$

$V_{ec} = V_e - V_c = 0 - 10V \Rightarrow \underline{V_{ec} = -10V}$

we assumed that

②. way

KVL @ group 1

$-V_{ae} + 16 - 12 + 4 + 6 = 0$

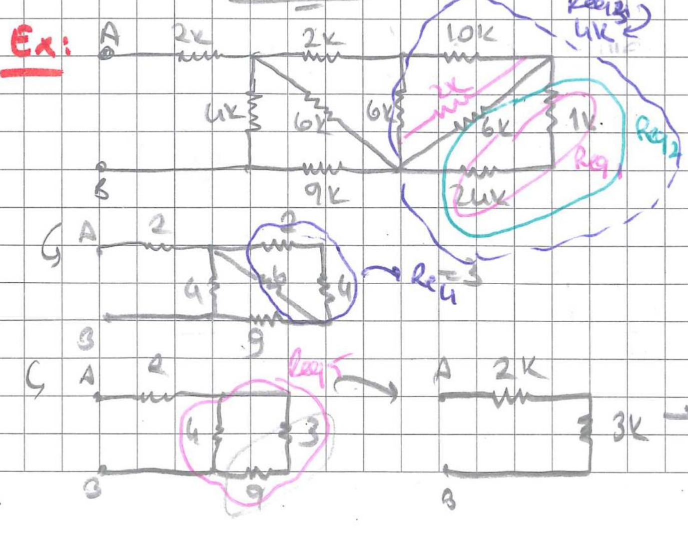
$\underline{V_{ae} = 14V}$

KVL @ group 2

$-V_{ec} + 10 - 24 + 16 - 12 = 0$

$\underline{V_{ec} = -10V}$

Ex:



$R_{AB} = ?$

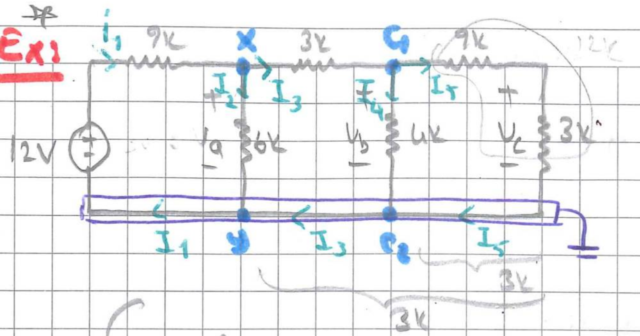
$R_{eq_1} = 1 + 2 = 3k\Omega \qquad R_{eq_2} = \frac{6 \times 3}{6+3} = 2k\Omega$

$R_{eq_3} = \frac{6 \times 12}{12+6} = 4k\Omega$

$R_{eq_4} = \frac{6 \times 6}{12} = 3k\Omega \qquad R_{eq_5} = \frac{4 \times 12}{12+4} = 3k\Omega$

$\rightarrow R_{AB} = 2 + 3 = \underline{5k\Omega}$

Ex1

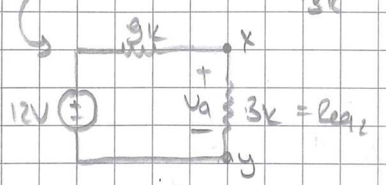


Find the all currents and voltages.

$9k + 3k = 12k$

$R_{eq_1} = \frac{1}{4} + \frac{1}{12} = 3k$ $\quad 3k + 3k = 6k$

$R_{eq_2} = \frac{1}{4} + \frac{1}{6} = 3k$



$V_a = (12V)\frac{3k}{12k} = 3V$

$V_b = (3V)\frac{3k}{6k} = 1,5V$

$V_c = (1,5V)\frac{3k}{12k} = 0,375V$

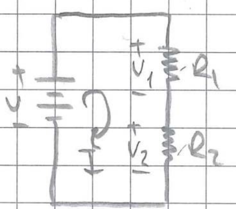
We use voltage division

$I_1 = \frac{12-3}{9} = 1mA$

$I_2 = \frac{3-0}{6} = 0,5mA$ $\quad I_4 = \frac{1,5-0}{4} = 0,375mA$

$I_3 = \frac{3-1,5}{3} = 0,5mA$ $\quad I_5 = \frac{0,375-0}{3} = 0,125mA$

* VOLTAGE DIVISION *



* All resistors in series and share same current.

By KVL & Ohm's Law

$0 = -V + V_1 + V_2 \rightarrow V = V_1 + V_2$

$V = IR_1 + IR_2 = IR_{eq}$

$V_1 = \frac{R_1}{R_1+R_2} \cdot V$

$V_2 = \frac{R_2}{R_1+R_2} \cdot V$

* The source voltage V is divided among the resistors in direct proportion to their resistance.

* → the larger resistance has larger voltage drop.

* This called the principle of voltage division and the circuit is called a voltage divider circuit.

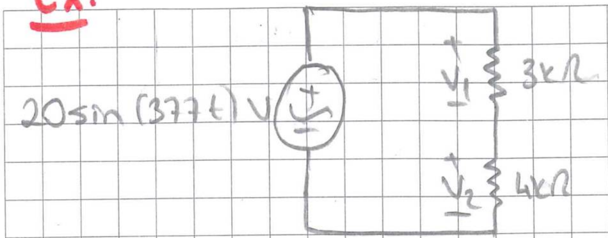
! RULE ⇒ $V_n = \frac{R_n}{R_1+R_2+\dots R_n} \cdot V_{Total} \Rightarrow \boxed{V_n = \left[\frac{R_n}{R_{eq}}\right] V_{Total}}$

Note: in real life red probe → + in circuits.
black probe → −

AC voltage } $V = A\sin(2\pi f + \theta)$

A → magnitude, f → frequency, θ → phase

Ex:

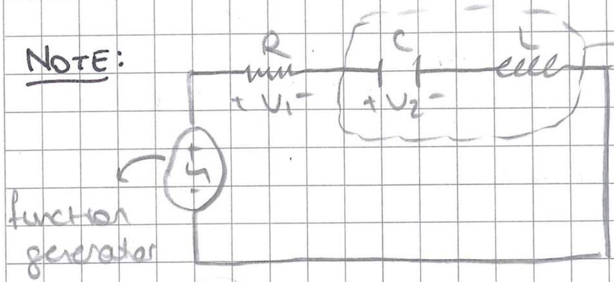


V_1 and V_2 = ?

$$V_1 = \frac{R_1}{R_1+R_2} \cdot V_T = 8{,}57 \sin(377t)\,V$$

$$V_2 = \frac{R_2}{R_1+R_2} \cdot V_T = 11{,}4 \sin(377t)\,V$$

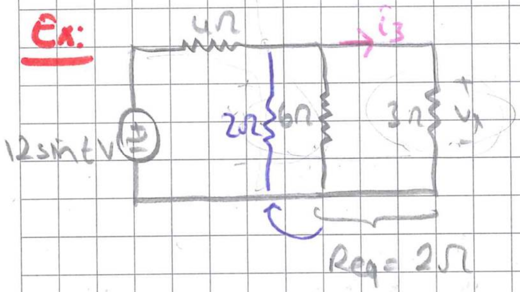
NOTE:



→ we will see the phase differences.

→ and these two elements energy storage in the circuit.

Ex:

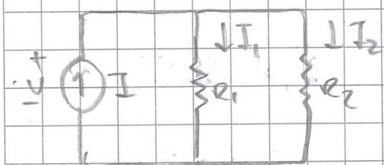


V_x = ? , i_3 = ?

$$V_x = \frac{2}{(4+2)}(12\sin t) = \underline{4\sin t\,V}$$

$$i_3 = \frac{4\sin t\,V}{3} = \underline{1{,}33\sin t\,A}$$

* CURRENT DIVISION *



All resistors in parallel and share the same voltage.

By KCL & Ohm's Law

$$0 = -I + I_1 + I_2 \rightarrow I = I_1 + I_2$$

$$I = \frac{V}{R_1} + \frac{V}{R_2}$$

$$I_1 = \frac{R_2}{R_1+R_2} \cdot I$$

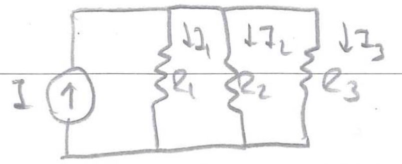
$$I_2 = \frac{R_1}{R_1+R_2} \cdot I$$

* The total current I is shared by resistors in inverse proportion to their resistors.

* → The smaller resistor has the larger current flow.

This is called the principle of current division and the circuit is called current divider circuit.

! RULE ⇒ $I_n = \dfrac{1/R_n}{\frac{1}{R_1} + \frac{1}{R_2} + \dots \frac{1}{R_n}} \cdot I_{Total}$ ⇒ $I_n = \left[\dfrac{R_{eq}}{R_n}\right] I_{Total}$



$$I_1 = \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3} \cdot I \qquad I_3 = \frac{R_1 \parallel R_2}{R_3 + R_1 \parallel R_2} \cdot I$$

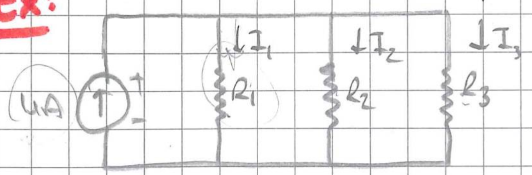
$$I_2 = \frac{R_1 \parallel R_3}{R_2 + R_1 \parallel R_3} \cdot I$$

* If we can use conductance (G)

$$I_n = \frac{G_n}{G_1 + G_2 + \dots G_n} \cdot I_T \Rightarrow \left[\frac{G_n}{G_{eq}}\right] \cdot I_{Total} = I_n$$

* the larger conductance value resistor has the larger current.

Ex:



$R_1 = 200\Omega$, $R_2 = 400\Omega$ and $R_3 = 600\Omega$

so find the currents I_1, I_2, I_3.

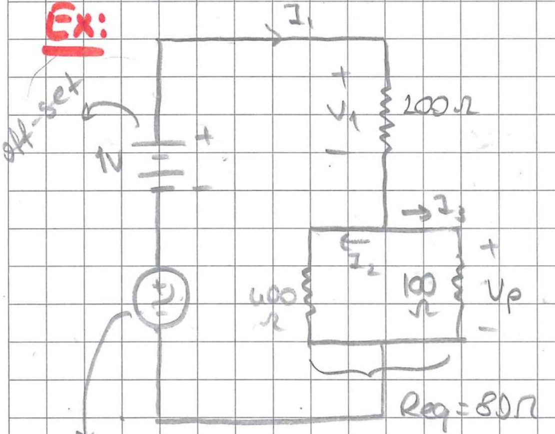
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = 109\Omega$$

$$I_1 = \frac{109}{200} \cdot 4 = \underline{2{,}18A}$$

$$I_2 = \frac{109}{400} \cdot 4 = \underline{1{,}09A}$$

$$I_3 = \frac{109}{600} \cdot 4 = \underline{0{,}722A}$$

Ex:

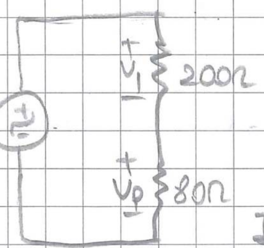


$0{,}5 \sin(20t) V$
(function generator)

$V_P, V_1, I_1, I_2, I_3 = ?$

$V_{Tot} = 0{,}5 \sin(20t) + 1V \rightarrow$ DC off-set voltage (DC Bayes)

$\rightarrow$



$$I_1 = \frac{1 + 0{,}5 \sin(20t)}{280\Omega}$$

$$\underline{I_1 = 3{,}57 + 1{,}79 \sin(20t) mA}$$

DC term AC term

$V_1 = I_1 \cdot R_1$

$V_1 = (3{,}57 + 1{,}79 \sin(20t))(200)$

$V_1 = 0{,}714 + 0{,}357 \sin(20t) V$

$\rightarrow$ or by voltage division

$$V_1 = V_T \cdot \frac{200}{280}$$

$$V_P = \frac{V_T \cdot 80\Omega}{280\Omega} = \underline{0{,}287 + 0{,}143 \sin(20t) V}$$

$$I_3 = \frac{V_P}{100\Omega} = \underline{2{,}86 + 1{,}43 \sin(20t) mA}$$

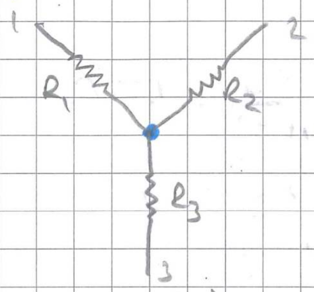
$$I_2 = \frac{V_P}{400\Omega} = \underline{0{,}714 + 0{,}357 \sin(20t) mA}$$

or

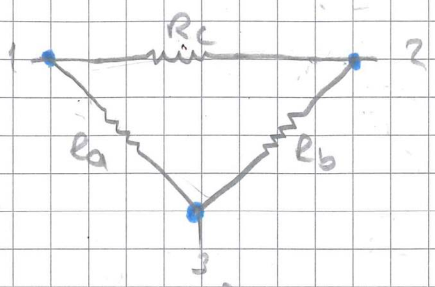
$$I_2 = \frac{R_3}{R_2 + R_3} \cdot I_1$$

$$I_2 = I_1 - I_3$$

* WYE & DELTA NETWORK * (3 terminals)



WYE(Y)



DELTA (Δ)

→ to transform $\Delta \rightarrow Y$

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

→ to transform $Y \rightarrow \Delta$

$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

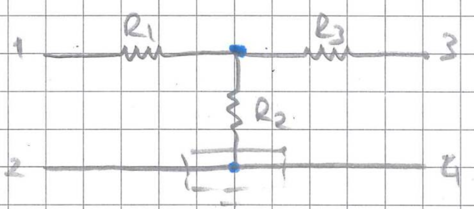
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

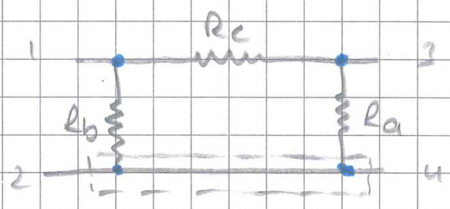
* if $R_1 = R_2 = R_3 = R \longrightarrow R_a = R_b = R_c = 3R$

$R_a = R_b = R_c = R \longrightarrow R_1 = R_2 = R_3 = 1/3R$

* T & Π NETWORKS * (4 terminals)



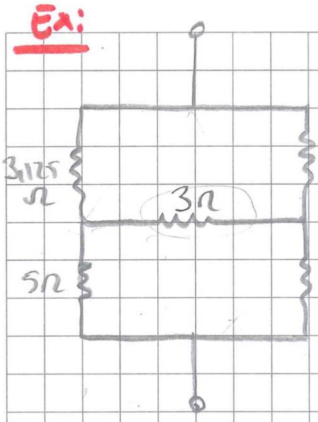
T

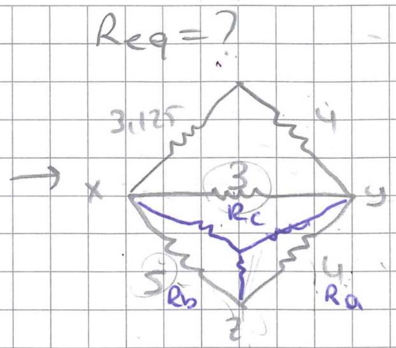


Π (PHI)

* we should use these transforms in distribution of power in stators and windings in motors/generators.

Ex:

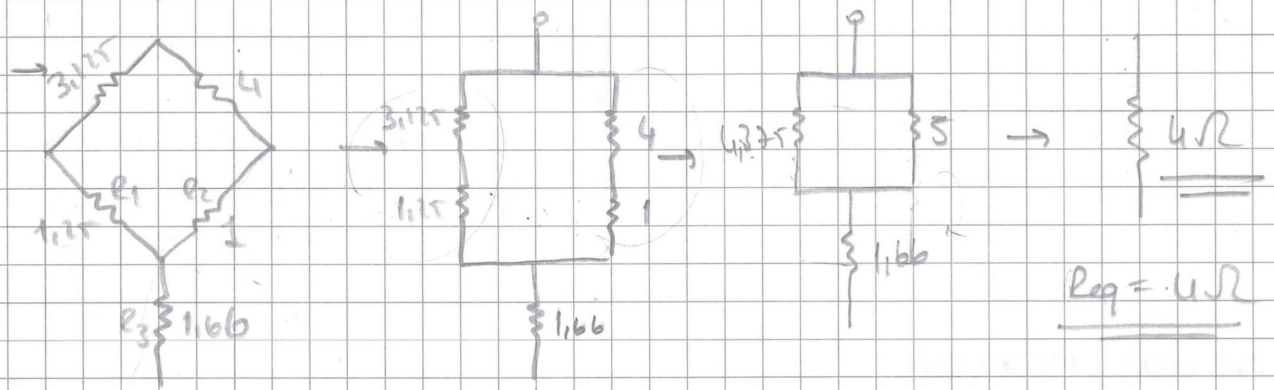




$R_1 = \frac{5 \cdot 3}{4+5+3} = 1,25\Omega$

$R_2 = \frac{4 \cdot 3}{4+5+3} = 1\Omega$

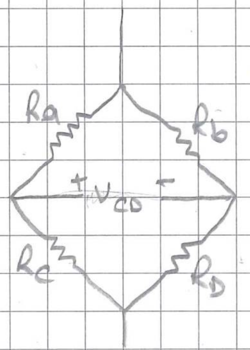
$R_3 = \frac{4 \cdot 5}{4+5+3} = 1,66\Omega$



Distribution of power in stators and windings in motors / generators.

↳ Wye windings provide better torque at low rpm and delta windings generates better torque at high rpm.

* BRIDGE CIRCUITS *



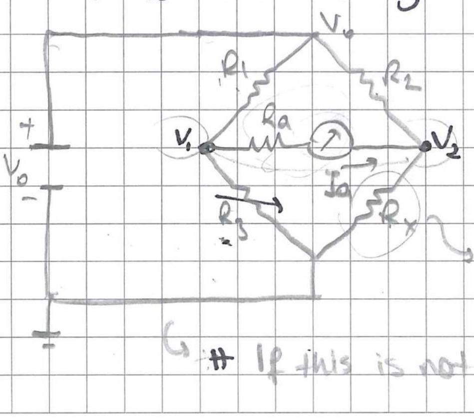
If $R_a = R_b = R_c = R_D$ then $V_{CD} = 0$

↳ We can use this in sensing and full-wave rectifier circuits.

$R_1 \cdot R_x = R_2 \cdot R_3$

$R_x = \frac{R_2 \cdot R_3}{R_1}$

Wheatstone Bridge Circuit



→ Measurement instrument based on differential measurement

* Balanced Condition: $I_a = 0$ ↳ $V_1 = V_2$ } $R_x = \left(\frac{R_2}{R_1}\right) R_3$ for balanced con.

↳ (could be sensor and can be controlled by bridge)

↳ # If this is not balanced then you need to turned Δ / Y.

NODAL & MESH ANALYSIS

→ Provide step-by-step instructions for nodal analysis, which is a method to calculate node voltages and currents that flow through component in circuit.

→ Provide step-by-step instructions for mesh analysis, which is a method to calculate voltage drops and mesh currents that flow around loops in circuit.

! **Remark:** $a_1x + b_1y = c_1$
$a_2x + b_2y = c_2$

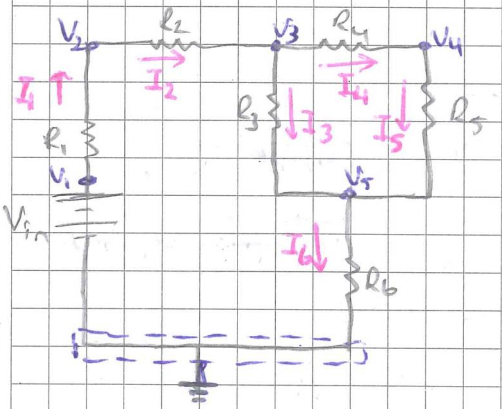
↳ Solution by Determinant

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} \qquad D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} \qquad \left.\begin{aligned} x &= \frac{D_x}{D} \\ y &= \frac{D_y}{D} \end{aligned}\right.$$

→ Nodal Analysis

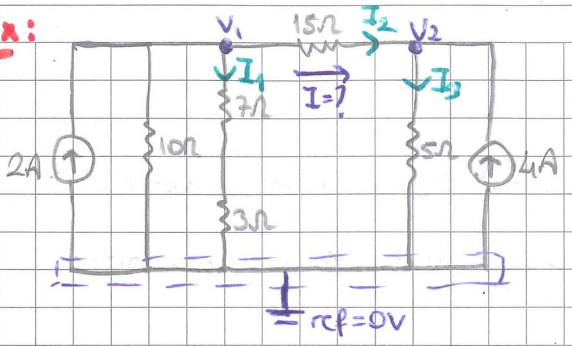


1) Define nodes and choose reference point.
2) Label the voltages at nodes
3) Label the currents flowing through at nodes
4) Use KCL
5) Use Ohm's Law to transform currents.
6) Solve equations.

One or more of the node voltages may have a negative sign. This depends on which node you choose as your reference node.

Node voltages must have a magnitude less than the sum of the voltage sources in the circuit.

Ex:



Determine the current flowing left to right through the 15Ω resistor by nodal analysis.

KCL @ node 1 $\rightarrow 2A = I_1 + I_3$

KCL @ node 2 $\rightarrow 4A + I_2 = I_3$

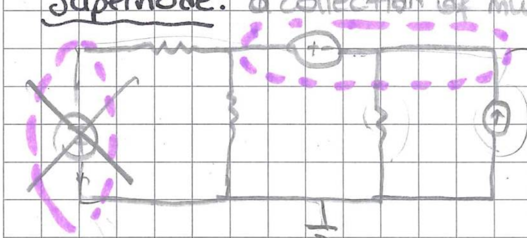
by Ohm's Law $\rightarrow 2_A = \frac{V_1}{10} + \frac{V_1 - V_2}{15} \Rightarrow 60 = 5V_1 - 2V_2$

$\frac{V_1 - V_2}{15} + 4A = \frac{V_2}{5} \Rightarrow 60 = -V_1 + 4V_2$

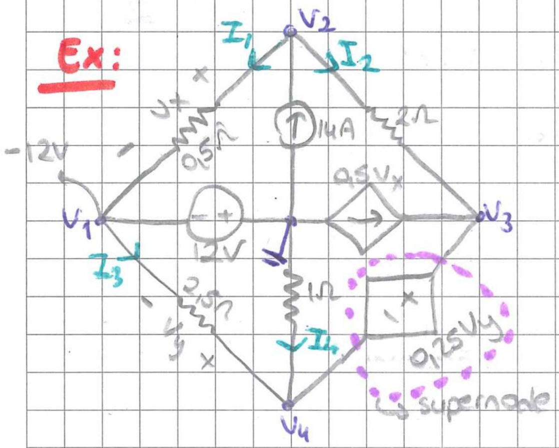
$V_1 = 20V$
$V_2 = 20V$

$I = \frac{V_1 - V_2}{15} = \underline{\underline{0A}}$

Supernode: a collection of multiple nodes separated by voltage sources.



Ex:



find V_1, V_2, V_3, V_4.

$\underline{V_1 = -12V}$ because the reference point.

KCL @ V_2 node $\rightarrow I_1 + I_4 = 14A \rightarrow \frac{V_2 - (-12)}{0.5} + \frac{V_2 - V_3}{2} = 14$

$\Rightarrow \underline{5V_2 - V_3 = 20}$

KCL @ supernode $\rightarrow I_2 + 0.5V_x + I_3 + I_4 = 0$

$\Rightarrow \frac{V_2 - V_3}{2} + 0.5(V_2 + 12) + \frac{(-12V) - V_4}{2.5} + (-V_4) = 0$

$\Rightarrow \underline{10V_2 - 5V_3 - 14V_4 - 12}$

Additional Eq $\rightarrow V_x = V_2 + 12$

$\rightarrow V_3 - V_4 = 0.12 V_y$

$V_4 - V_1$

$\Rightarrow \underline{V_3 - 1.2V_4 = 2.4}$

$$\begin{bmatrix} 5 & -1 & 0 \\ 10 & -5 & -14 \\ 0 & 1 & -1.2 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} -20 \\ -12 \\ -2.4 \end{bmatrix}$$

we solved in Matlab

$\underline{V_2 = -4V}$ $\underline{V_3 = 0V}$ $\underline{V_4 = -2V}$

if we solved by hand

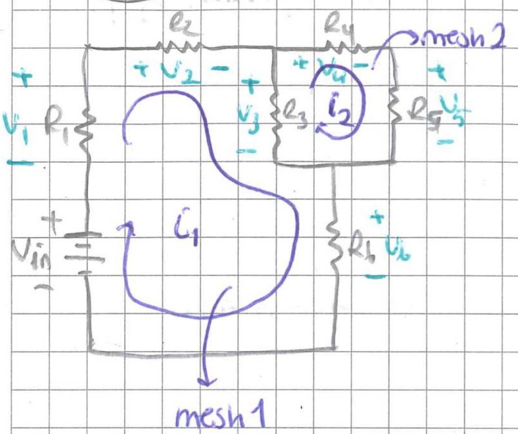
$$D = \begin{vmatrix} 5 & -1 & 0 & 5 & -1 \\ 10 & -5 & -14 & 10 & -5 \\ 0 & 1 & -1.2 & 0 & 1 \end{vmatrix} = (30+0+0) - (12-70+0) = 88$$

$$D_{V_2} = \begin{vmatrix} -20 & -1 & 0 & -20 & -1 \\ -12 & -5 & -14 & -12 & -5 \\ -2.4 & 1 & -1.2 & -2.4 & 1 \end{vmatrix} = -352$$

$V_2 = \frac{D_{V_2}}{D} = \underline{\underline{-4V}}$

Mesh Analysis

Mesh → small loops in circuits



1) Define all meshes in the circuit.

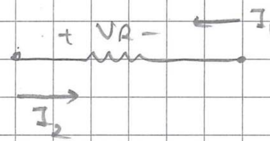
2) label the current flowing in each mesh

3) label the voltage across each component

4) use KVL

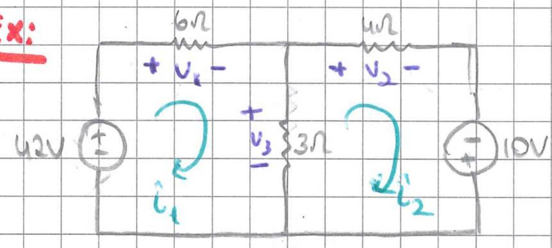
5) use Ohm's law to transform voltages

6) Solve equations.



$V_R = (I_1 - I_2)R$

Ex:



find the current flows on R_3, and V_3

KVL @ 1^{st} mesh → $-42 + V_1 + V_3 = 0$

$\Rightarrow -42 + 6i_1 + (i_1 - i_2)3 = 0$

$9i_1 - 3i_2 = 42$

KVL @ 2^{nd} mesh

→ $-V_3 + V_2 - 10V = 0$

$\Rightarrow -(i_1 - i_2)3 + 4i_2 - 10 = 0$

$-3i_1 + i_2 = 10$

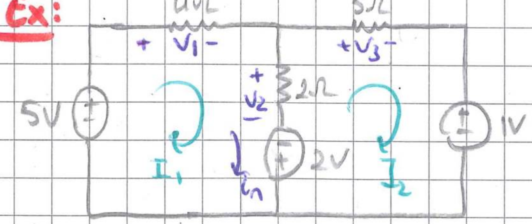
$i_1 = 6A$

$i_2 = 4A$

$i_{R_3} = i_1 - i_2$

$i_{R_3} = 2A$ → $V_3 = 2 \cdot 3 = 6V$

Ex:



Determine the power supplied by the 2V source by use Mesh analysis.

$I_n = (I_1 - I_2) = 1,23A$

$P_{2V} = (-)(2V)(1,23)$

$= -2,47W$

KVL @ 1^{st} mesh

$-5V + V_1 + V_2 - 2V = 0$

$-7V + 4I_1 + 2(I_1 - I_2) = 0$

$6I_1 - 2I_2 = 7V$

KVL @ 2^{nd} mesh

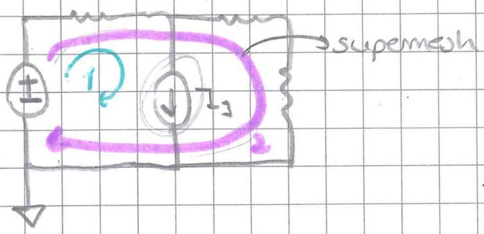
$2V - V_2 + V_3 + 1V = 0$

$3V - 2(I_1 - I_2) + 5I_2 = 0$

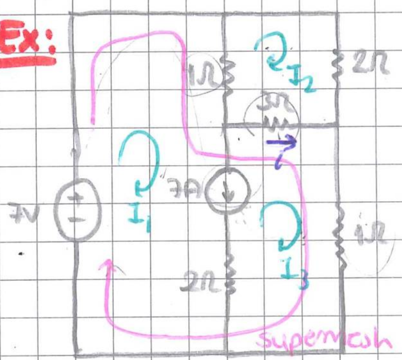
$-2I_1 + 7I_2 = -3V$

$I_1 = 1,132A, \quad I_2 = -0,1053A$

Supermesh: A mesh that contains multiple meshes with a shared current source.



Ex:



Determine the current i by using Mesh Analysis.

KVL @ supermesh

$$-7+(I_1-I_2)+3(I_3-I_2)+(I_3)=0$$

$$I_1-4I_2+4I_3=7 \quad (1)$$

KVL @ mesh 2

$$(I_2-I_1)+2I_2+3(I_2-I_3)=0$$

$$-I_1+6I_2-3I_3=0 \quad (2)$$

$$i_1-i_3=7A \quad (3)$$

↳ independent current source is related to mesh current

$$D=\begin{bmatrix}1&-4&4\\-1&6&-3\\1&0&-1\end{bmatrix}\cdot\begin{bmatrix}I_1\\I_2\\I_3\end{bmatrix}=\begin{bmatrix}7\\0\\7\end{bmatrix} = -14$$

$$D_{I_1}=\begin{bmatrix}7&-4&4\\0&6&-3\\7&0&-1\end{bmatrix}=-126$$

$$D_{I_2}=\begin{bmatrix}1&7&4\\-1&0&-3\\1&7&-1\end{bmatrix}=-35$$

$$D_{I_3}=\begin{bmatrix}1&-4&7\\-1&6&0\\1&0&7\end{bmatrix}=-28$$

$$I_1=\frac{D_{I_1}}{D}=9A$$

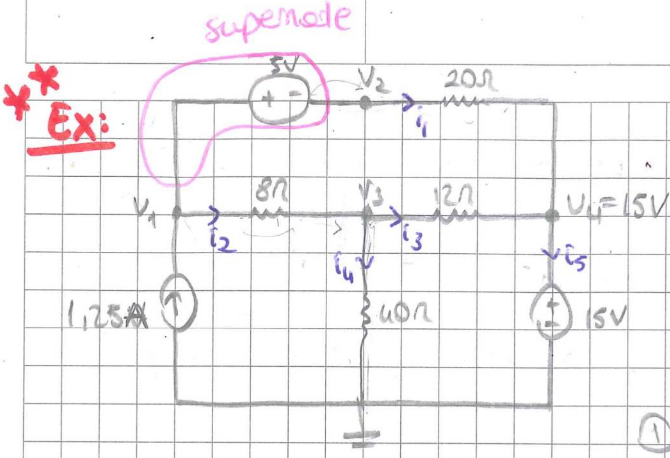
$$I_2=\frac{D_{I_2}}{D}=2{,}5A$$

$$I_3=\frac{D_{I_3}}{D}=2A$$

$$i=I_3-I_2$$

$$i=-0{,}5A$$

Ex:



Find the V_1, V_2, V_3, V_4 by using Nodal Analysis.

KCL @ supernode

$1{,}25A = i_1 + i_2$

$1{,}25 = \frac{V_2 - 15}{20} + \frac{V_1 - V_3}{8}$

(1) $5V_1 + 2V_2 - 5V_3 = 80$

KCL @ V_3

$i_2 = i_3 + i_4$

$\frac{V_1 - V_3}{8} = \frac{V_3 - 15}{12} + \frac{V_3}{40}$

(2) $15V_1 - 28V_3 = -150$

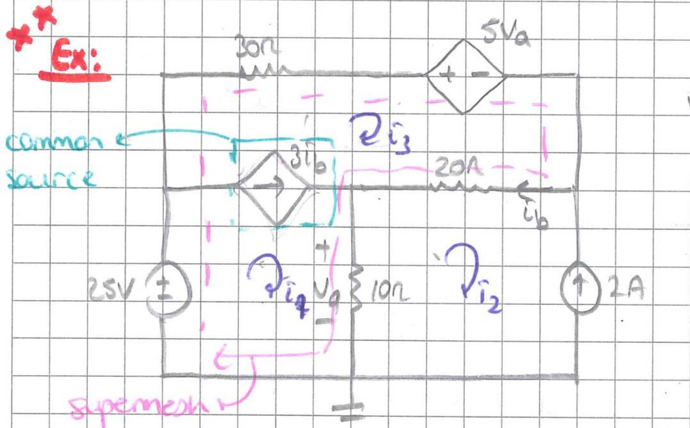
By using the supernode

(3) $V_2 - V_1 = 5$

$$\underbrace{\begin{bmatrix} 5 & 2 & -5 \\ 15 & 0 & -28 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 80 \\ -150 \\ 5 \end{bmatrix}}_{\text{solve by Matlab}}$$

$\underline{V_1 = 22{,}4V}$ $\underline{V_2 = 27{,}4V}$ $\underline{V_3 = 17{,}4V}$ $\underline{V_4 = 15V}$

Ex:



Find i_1, i_2, i_3 by using Mesh Analysis

KVL @ supermesh

$-25V + 30i_3 + 5V_a + 20(i_3 - i_2) + 10(i_1 - i_2) = 0$

$50i_3 + 60i_1 - 80\overset{2A}{i_2} = 25$

$60i_1 + 50i_3 = -135$ (1)

$V_a = 10(i_1 - i_2)$

$i_2 = -2A$ (3)

$i_b = i_3 - i_2 = i_3 + 2$

Use dependent current source

$i_1 - i_3 = 3i_b \rightarrow (i_3 - i_2)$

$i_1 - 4i_3 = 6$ (2)

if we solved (1) and (2) equations together

$\underline{i_1 = 12{,}8A}$ $\underline{i_2 = -2A}$ $\underline{i_3 = -1{,}7A}$

→ Or if we want to solved by matrices

$$\begin{bmatrix} 60 & -80 & 50 \\ 1 & 0 & -4 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 25 \\ 6 \\ -2 \end{bmatrix}$$

* LINEAR SYSTEMS *

$x \rightarrow$ [linear circuit] $\rightarrow y = f(x)$

- if x is doubled $\rightarrow y = f(2x) = 2f(x)$
- if x is multiplied by a constant $\underline{a}$ $\rightarrow y = f(\underline{a}x) = \underline{a}f(x)$
- if $x = x_1 + x_2 \rightarrow y = f(x) = f(x_1 + x_2) = f(x_1) + f(x_2)$

then the system is linear

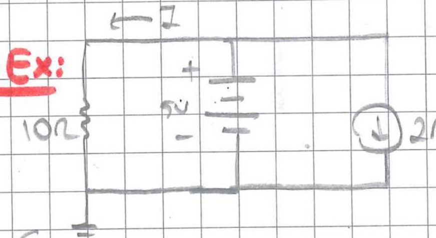
NOTE: $P = V \cdot I$
$= I^2 R$
$= \dfrac{V^2}{R}$

power has non-linear relation with current and voltage

linear components $\rightarrow$ resistor
$\rightarrow$ capacitor
$\rightarrow$ inductor
$\rightarrow$ indep. sources

non-linear components $\rightarrow$ diodes
$\rightarrow$ transistor
$\rightarrow$ SCRs
$\rightarrow$ magnetic switches

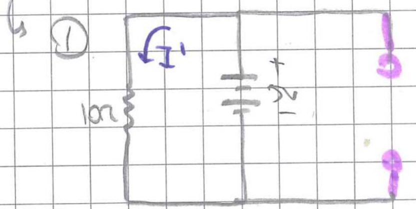
Ex:

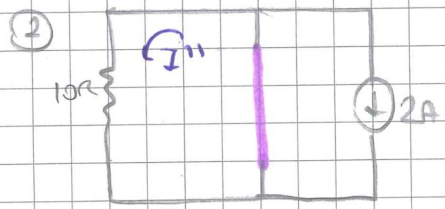


$I = ?$

* this circuit can be seperated into two circuits and

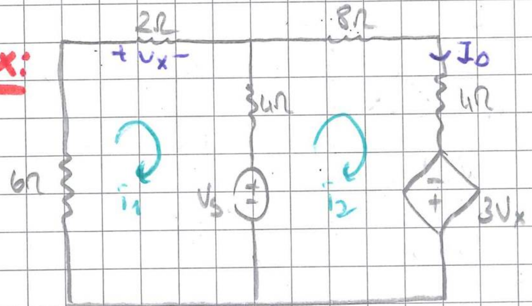
$I = I' + I''$





$I' = 5V / 10\Omega = 0{,}5\,A \quad + \quad I'' = 0\,A \quad = \rightarrow \quad \underline{I = 0{,}5A}$

Ex:



$I_0 = ?$ when $\underline{V_s = 12}$ & $\underline{V_s = 24V}$

$\rightarrow$ By Mesh Analysis

KVL @ Mesh 1

$6i_1 + 2i_1 + 4(i_1 - i_2) + V_s = 0$

KVL @ Mesh 2

$\underbrace{-V_s + 4(i_2 - i_1) + 8i_2 + 4i_2 - 3(V_x) = 0}_{i_1 = -6i_2}$, $V_x \rightarrow 2i_1$

$i_0 = i_2 \quad , \quad i_2 = \dfrac{V_s}{76}$

@ $V_s = 12V \quad i_0 = \dfrac{12}{76}$

@ $V_s = 24V \quad i_0 = \dfrac{24}{76}$

* this system is linear system because if we doubled input, the output doupled too.

If your circuit is linear, your current sources addition must be equal to total current source:

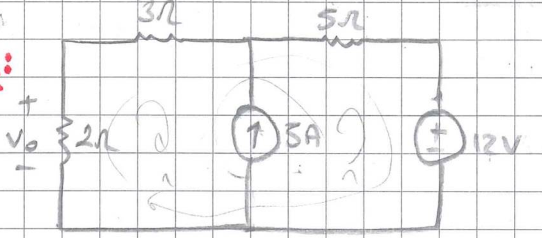
* SUPERPOSITION *

→ Separating the contributions of the DC and AC independent sources.

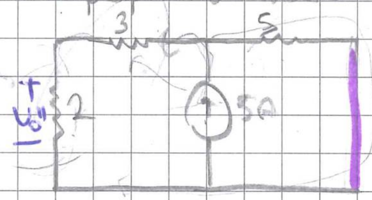
① Voltage sources should be replaced with short circuits.

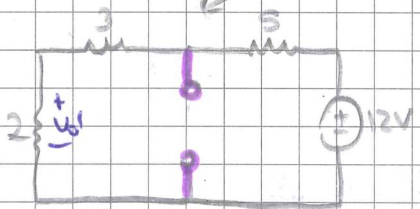
② Current sources should be replaced with open circuits.

Ex:



Use the superposition theorem to find V_o.





$V_o' = (12)\left(\frac{2}{10}\right)$

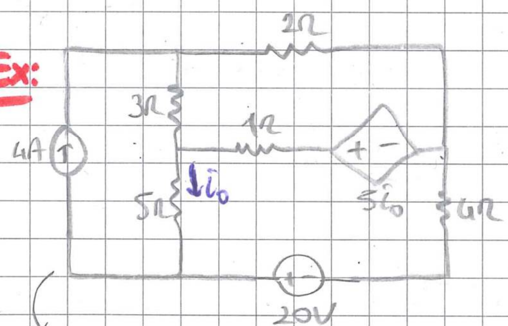
$V_o' = 2{,}4V$

$V_o'' = I_o \cdot 2 = \frac{5A \cdot 5\Omega}{10} \cdot 2 = 5V$

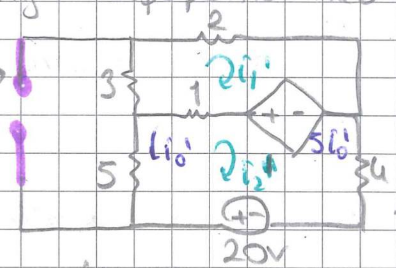
$I_o = \frac{5 \cdot 5}{10}$

$V_o = V_o' + V_o'' = \underline{7{,}4V}$

Ex:



Using the superposition theorem find i_o.



$3i_1' + 2i_1' - 5i_o' + (i_1' - i_2') = 0$

$6i_1' + 4i_2' = 0$

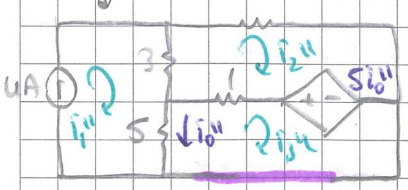
$-20 + 5i_2' + i_2' - i_1' + 5i_o' + 4i_2' = 0$

$-i_1' + 5i_2' = 20$

$i_o' = -i_2'$

$i_2' = 3{,}53A$

$i_o' = -3{,}53A$



$i_1'' = 4A$

$i_o'' = 4 - i_3''$

$2i_2'' - 5i_o'' + i_2'' - i_3'' + 3(i_2'' - 4) = 0$

$6i_2'' + 4i_3'' = 32$

$i_3'' - i_2'' + 5i_o'' + 4i_3'' + 5(i_3'' - 4) = 0$

$5i_3'' - i_2'' = 0$

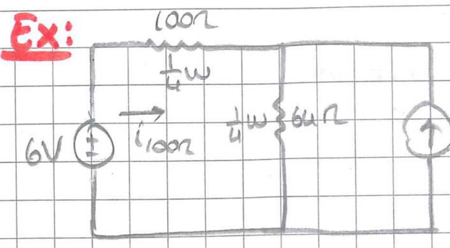
$i_3'' = 0{,}94A$

$i_o'' = 3{,}06A$

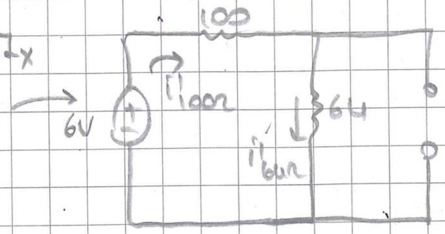
$i_o = i_o' + i_o''$

$= \underline{-0{,}47A}$

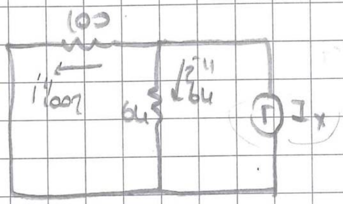
Ex:



Find the maximum value of I_x



$\left.\begin{array}{l} i'_{100\Omega} \\ i'_{64\Omega} \end{array}\right\}$ same $= \dfrac{6}{164} = \underline{37mA}$



$i''_{100} = \dfrac{64\Omega}{164\Omega} \cdot I_x$

$i''_{64} = \dfrac{100\Omega}{164\Omega} I_x$

→ Power rate $= \frac{1}{4}W$, in the initial circuit;

$i_{100\Omega} \leq \sqrt{\dfrac{0{,}25W}{100\Omega}} \Leftarrow P_{100\Omega} = (i_{100})^2 100 \leq 0{,}25W$

$i_{64} \leq \sqrt{\dfrac{0{,}25W}{64\Omega}} \Leftarrow P_{64\Omega} = (i_{64})^2 \cdot 64 \leq 0{,}25W$

a) The maximum power of $i_{100\Omega} \leq 50mA$

b) The maximum power of $i_{64\Omega} \leq 62{,}5mA$

in the original circuit; $i_{64\Omega} = i'_{64} + i''_{64}$

$i_{100\Omega} = \underbrace{i''_{100} - i'_{100}}$ because the direction of current flows

a-1) from eq a) $(\overset{\frac{64}{164}I_x}{i''_{100}} - \overset{37mA}{i'_{100}}) \leq 50mA \Rightarrow I_x \leq (87mA)\dfrac{164}{64}$

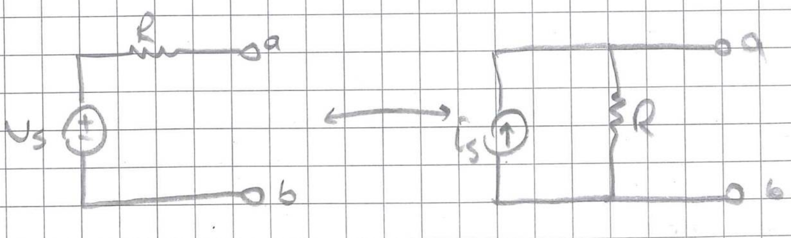
b-1) from eq b) $(\underset{37mA}{i'_{64}} + \underset{\frac{100}{164}I_x}{i''_{64}}) \leq 62{,}5mA \Rightarrow I_x \leq (25mA)\dfrac{164}{100}$

$\left.\begin{array}{l} \text{a-1)} \cdot I_x \leq 222{,}9\,mA \\ \text{b-1)}\ I_x \leq 41\,mA \end{array}\right\}$ It needs to satisfy both of them so;

The final $\underline{I_x \leq 41mA}$

* SOURCE TRANSFORMATION *

→ is the process of replacing a voltage source V_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.



$V_s = i_s R$

$i_s = \dfrac{V_s}{R}$

source transformation can apply on dependent sources.

for voltage sources → ideal: An ideal voltage source has no internal resistance

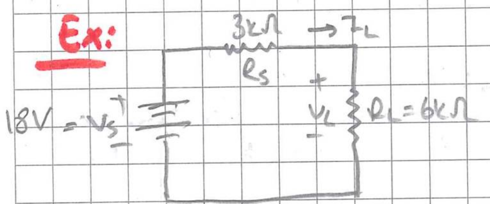
→ real: A real voltage source modeled as an ideal voltage source in series with a resistor. → (limiter)

for current sources → ideal: An ideal current source has no internal resistance.

→ real: A real current sources is modeled as an ideal current source in parallel with a resistor. → (limiter)

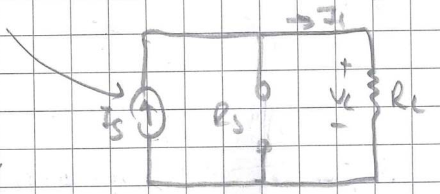
Applies for both voltage and current sources } $R = 0\Omega \rightarrow P = 0W$
$R = \infty\Omega \rightarrow P = 0W$

Ex:



→ Find an equivalent current source to replace V_s and R_s in the circuit

→ Find I_L and V_L



$I_s = I_L$

$V_L = 2mA(6k\Omega) = 12V$

$P_L = 12V \cdot 2mA = 24mW = P_{I_s}$

$V_L = \frac{R_L}{R_L + R_s} V_s$

$V_L = \frac{6k\Omega}{6k\Omega + 3k\Omega} 18 = 12V$

$I_L = V_L / R_L = 2mA$

$P_{V_s} = P_L + P_{R_s} = 12 \cdot (2mA) + (18-12)(2mA)$
$= 36mW$

→ if $R_s = 20k\Omega$

$I_s = \frac{R_L + R_s}{R_s} I_L = 2.67mA$

$V_L = 12V \rightarrow P_{I_s} = 32\ mW$

→ if $R_s = 3k\Omega$

$I_s = \frac{R_L + R_s}{R_s} I_L = 6mA$

$V_L = 12V \rightarrow P_{I_s} = 72mW$

→ if $R_s = 6k\Omega$

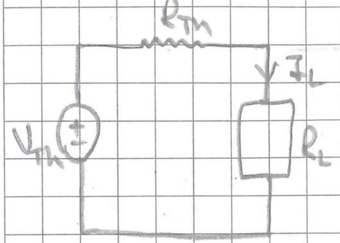
$I_s = \frac{R_L + R_s}{R_s} I_L = 4mA$

$V_L = 12V \rightarrow P_{I_s} = 48mW$

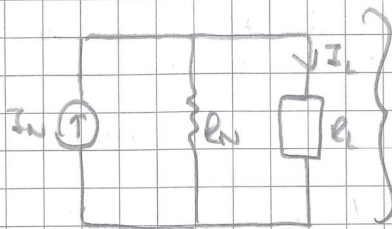
$\boxed{\downarrow R_s,\ \uparrow P_{I_s}}$ → but absorbed power is the same (kaynaktan daha fazla enerji çekiyor ama harcanan enerji aynı)

ideal source → @ $R_s = R_L$ condition

* THÉVENIN and NORTON EQUIVALENT *



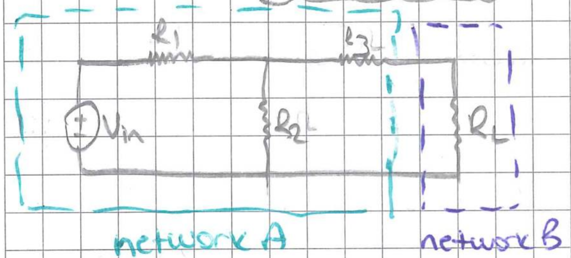
Thévenin equivalent



Norton equivalent

$I_L = \dfrac{V_{Th}}{R_{Th}+R_L}$ $\quad V_{Th} = I_N \cdot R_{Th}$ $\qquad I_N = \dfrac{V_{Th}}{R_{Th}}$ $\quad R_N = R_{Th}$

→ We have 4 approaches to find Thévenin Equivalent.



1th approach: Use repeated source transformations to arrive at a single voltage source in series with a single series resistance.

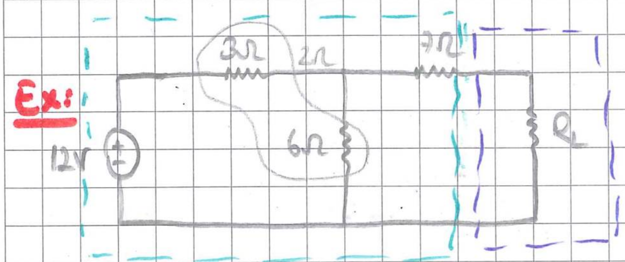
2nd approach: Open the load and determine the open-circuit voltage (V_{oc}), then short the load and determine short-circuit current (I_{sc})

3th approach: Open the load and determine the open-circuit voltage (V_{oc}) then deactive all independent sources and find the R_{eq}.

We use only dependent sources ← 4th approach: Open the load and determine the open-circuit voltage (V_{oc}), then deactivate all independent sources and apply test source (V_T)

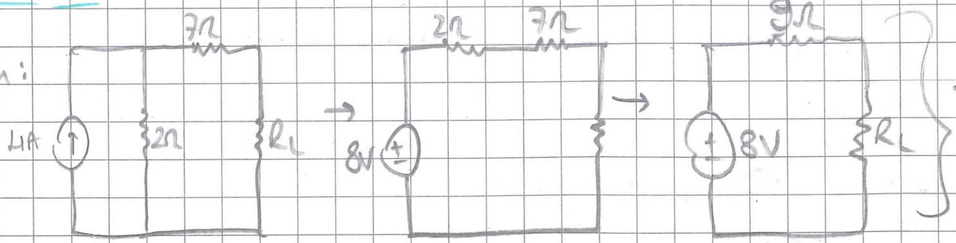
$V_{Th} = V_{oc}$ $\qquad V_T = R_{Th} \cdot I_{Test}$

Ex:



Find the Thevenin equivalent of this circuit, find the maximum power.

→ by the 1st approach:

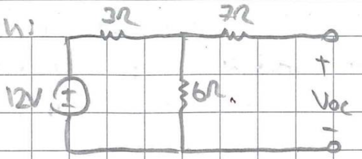


Thevenin eq.

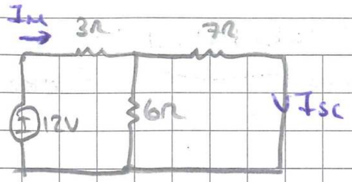
$P_{max} \Rightarrow P_L = \left(\dfrac{8}{9+R_L}\right)^2 R_L$

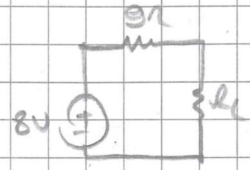
$8V = V_{Th}$
$9Ω = R_{Th}$

→ by the 2nd approach:



→





Thevenin eq.

$V_{oc} = 12\left(\frac{6}{3+6}\right)$

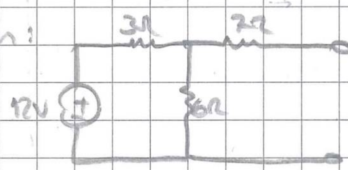
$V_{oc} = \underline{8V = V_{Th}}$

$I_M = \frac{12}{3+7\|6} = 1{,}92A$ (total I)

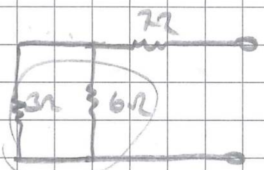
$I_{sc} = \frac{1{,}92 \times 6}{13} = 0{,}89A$

$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{8}{0{,}89} = \underline{9\Omega}$

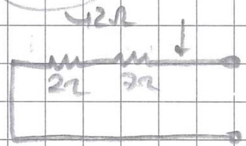
→ by the 3th approach:



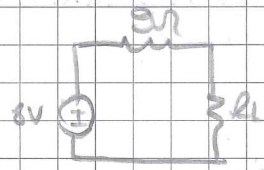
→



← $Req = R_{Th}$



→ $\underline{R_{Th} = 9\Omega}$

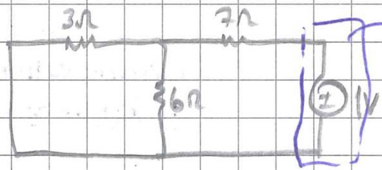


Thevenin eq

$V_{oc} = 12\left(\frac{6}{3+6}\right)$

$V_{oc} = \underline{8V = V_{Th}}$

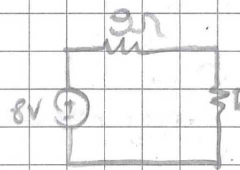
→ by the 4th approach:



→ test voltage ($V_T = 1V$)

$I_{Test} = \frac{1V}{9\Omega} = 0{,}11A$

$R_{Test} = \frac{V_T}{I_T} = \underline{9\Omega}$

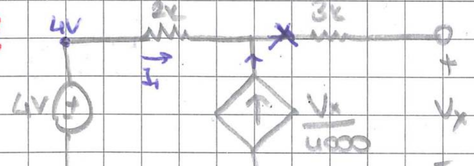


Thevenin eq.

$V_{oc} = 12\left(\frac{6}{3+6}\right)$

$V_{oc} = \underline{8V = V_{Th}}$

Ex:



Find the V_x and thevenin equivalent of the circuit

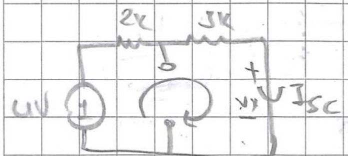
by the 2nd approach → $V_{oc} = V_{Th} = V_x$

→ if we apply KCL

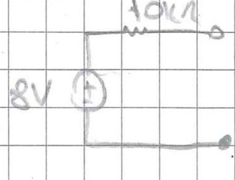
$\frac{4V - V_x}{2000} + \frac{V_x}{4000} = 0 \rightarrow \underline{V_x = 8V}$

$R_{Th} = \frac{V_{oc}}{I_{sc}}$

$= \frac{8V}{0{,}8mA} = 10k\Omega$

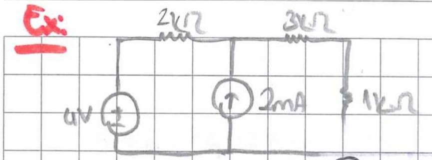


$I_{sc} = \frac{4V}{5k\Omega} = 0{,}8mA$



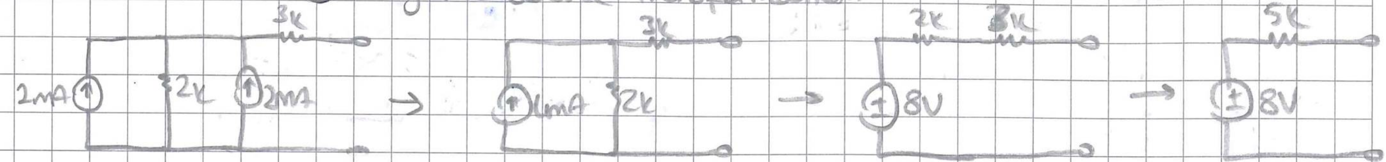
Thevenin eq.

Ex:



Determine the Thevenin and Norton eq. circuits for the network faced by the 1k resistor.

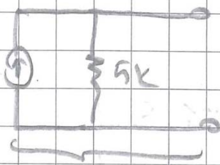
① → by the source transformation:



② → by the 3^{th} approach:

→ find the V_{oc}, find R_{Th} when all the indep. sources are killed

$I_{sc} = 1{,}6mA$

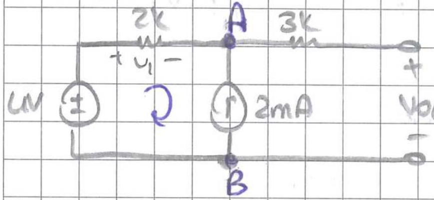


Norton Eq.

$R_{Th} = 5k\Omega$

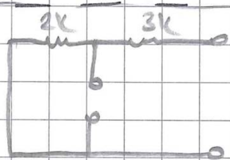
$V_{Th} = 8V$

Thevenin Eq.



$V_{oc} = V_{AB} = V_{Th}$

→ to find R_{Th}



← $R_{Th} = R_{eq}$

KVL → $-4V + V_1 + V_{oc} = 0$

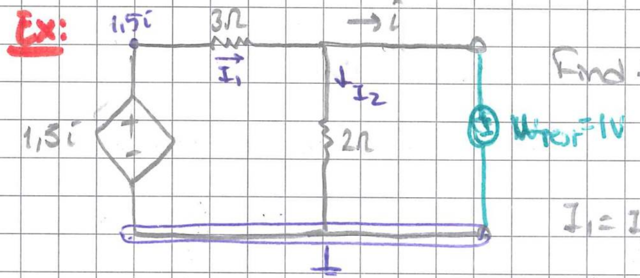
$V_1 = -4V$

$-4V - 4V + V_{oc} = 0$

$\underline{V_{oc} = 8V = V_{Th}}$

$\underline{R_{eq} = 5k}$

Ex:



Find the thevenin equivalent of this circuit.

by the 4^{th} approach $V_{Th} = 0 \rightarrow R_{Th} = ?$

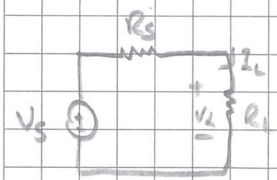
$I_1 = I_2 + i$

$R_{Th} = \dfrac{V_{Test}}{I_{Test}}$

$\dfrac{1{,}5i - 1}{3} = \dfrac{1}{2} + i \rightarrow i = -\dfrac{5}{3}A \quad I_{Test} = \dfrac{5}{3}A \quad R_{Th} = \dfrac{1}{5/3} = \dfrac{3}{5}$

$\underline{R_{Th} = 0{,}6\Omega}$

→ Power from a Practical Source :



$P_L = I_L \cdot V_L = \dfrac{V_s^2 R_L}{(R_s + R_L)^2}$

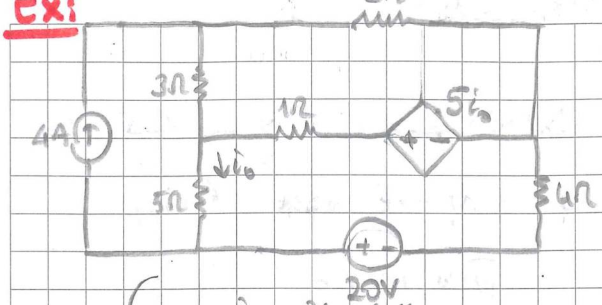
* Maximum power is delivered to the load when the load resistance (R_L) is equal to the Thevenin resistance (R_{Th}) of the circuit.

If $R_L = R_s \rightarrow \dfrac{V_s^2}{4R_L}$

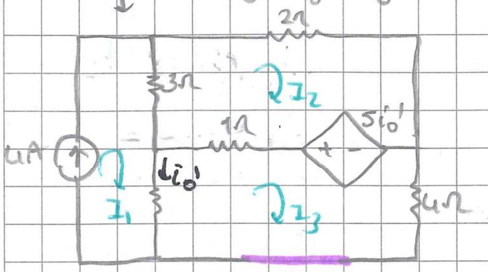
$\dfrac{d}{dR_L} P_L = 0 \rightarrow$ for the max power

↳ (türevin 0 olduğu nokta max)

Ex:



$i_o = i_o' + i_o''$



→ by the Mesh Analysis

$I_1 = 4A$, $i_o' = I_1 - I_3$

→ @ mesh 2

$3(I_2 - \overset{4}{I_1}) + 2I_2 - 5i_o' + (I_2 - I_3) = 0$

① $6I_2 + 4I_3 = 32$

→ @ mesh 3

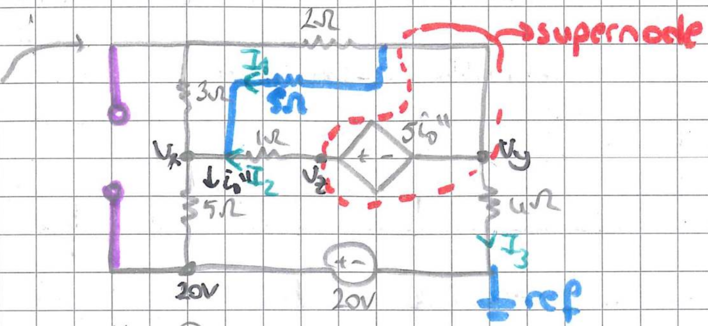
$(I_3 - I_2) + 5i_o' + 4I_3 + 5(I_3 - I_1) = 0$

② $I_2 + 5I_3 = 0$

$I_2 = 4.7A$
$I_3 = 0.94A$
$I_1 = 4A$

$i_o' = I_1 - I_3 = \boxed{3.06A}$

Find i_o in the circuit with superposition



→ KCL @ V_x

$I_1 + I_2 - i_o'' = 0$

$\frac{V_y - V_x}{5} + \frac{V_z - V_x}{1} = \frac{V_x - 20}{5}$ → $7V_x - V_y - 5V_z = 20$ ①

→ KCL @ supernode

$I_1 + I_2 + I_3 = 0$

$\frac{V_y - V_x}{5} + \frac{V_z - V_x}{1} + \frac{V_y}{4} = 0$ → $-24V_x + 9V_y + 20V_z = 0$ ②

→ @ supernode $V_z - V_y = 5i_o''$

$V_x + V_y - V_z = 20$ ③

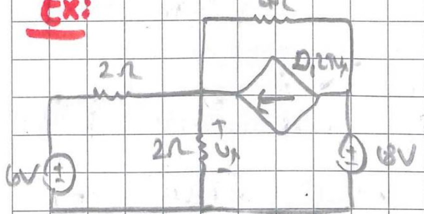
$$\begin{bmatrix} 7 & -1 & -5 \\ -24 & 9 & 20 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 20 \end{bmatrix}$$

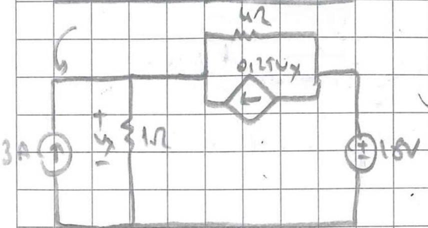
$V_x = 2.35V$
$V_y = 14.12V$
$V_z = -3.53V$

$i_o'' = \frac{V_x - 20}{5} = \boxed{-3.53A}$

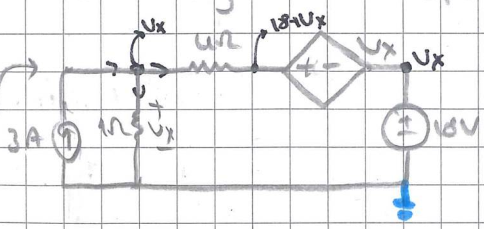
$i_o = i_o' + i_o'' = \underline{-0.47A}$

Ex:





Find the V_x using source transformation



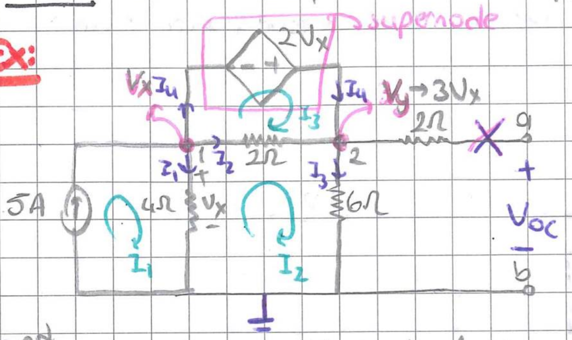
KCL @ V_x

$3A = \frac{V_x}{1} + \frac{(V_x - 18 - V_x)}{4}$

$\underline{V_x = 7.5V}$

NOTE: Source transformation can apply only in series sources.

Ex:



Find the Thévenin equivalent of this circuit.

1st way → by the Nodal Analysis

$V_y = V_x + 2V_x$ @ supernode

$V_y = 3V_x$

KCL @ node 1

$5A = I_1 + I_2 + I_4$

KCL @ node 2

$I_2 + I_4 = I_3$

$I_4 = I_3 - I_2$

$5 = I_1 + I_3$

$5 = \frac{V_x}{4} + \frac{3V_x}{6}$

$V_x = \frac{20}{3}$

$V_{oc} = 3V_x$

$\underline{V_{oc} = 20V}$

2nd way → by the Mesh Analysis

$I_1 = 5A$

@ 2nd Mesh

$V_x = 4(I_1 - I_2)$

$4(I_2 - I_1) + 2(I_2 - I_3) + 6I_2 = 0$

① $12I_2 - 2I_3 = 20$

@ 3th Mesh

$-2V_x + 2(I_3 - I_2) = 0$

② $6I_2 + 3I_1 = 40$

$I_2 = 3{,}3A$

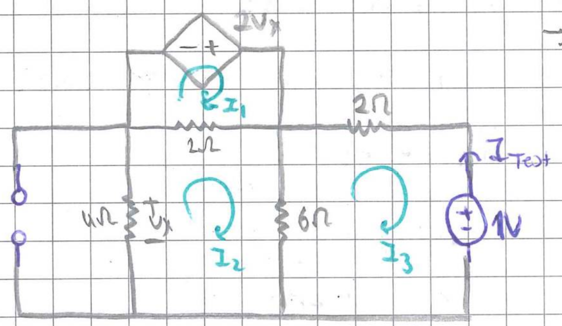
$I_3 = 10A$

$V_{oc} = V_{Th} = V_{ab} = I_2 \cdot 6$

$\underline{V_{oc} = 20V}$

after we find V_{oc}, by the 4th approach (adding test source)

→ we killed all the independent sources



by Mesh Analysis

$V_x = -4I_2$

@ 1st Mesh

$-2V_x + 2(I_1 - I_2) = 0$

① $2I_1 + 6I_2 = 0$

@ 2nd Mesh

$V_x + 2(I_2 - I_1) + 6(I_2 - I_3) = 0$

② $-2I_1 + 12I_2 - 6I_3 = 0$

@ 3th Mesh

$6(I_3 - I_2) + 2I_3 + 1 = 0$

③ $-6I_2 + 8I_3 = -1$

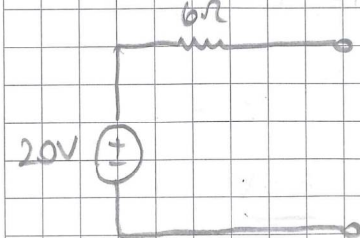
$$\begin{bmatrix} 2 & 6 & 0 \\ -2 & 12 & -6 \\ 0 & -6 & 8 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$I_3 = -1/6\ A$

$I_{Test} = -I_3$

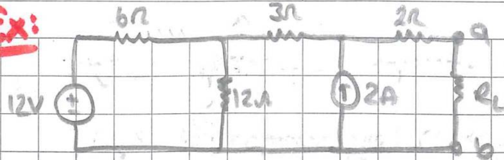
$I_{Test} = 1/6\ A$

$R_{Th} = \frac{V_{Th}}{I_{Test}} = \frac{1}{1/6} = \underline{6\Omega}$



Thévenin Equivalent Circuit

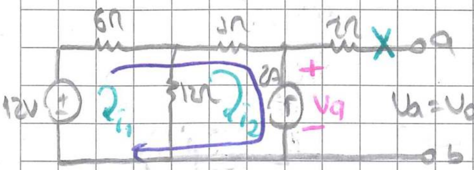
Ex:



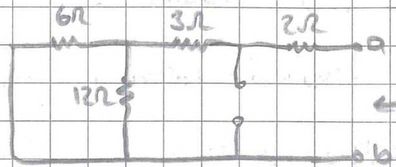
Find the value of R_L for max power transfer in the circuit, and find the maximum power

for the max power $R_L = R_{Th}$ → by the 3th approach (kill all dep. sources to find R_{Th})

→ Remove R_L and find V_{Th}



$V_a = V_{oc} = V_{Th}$



← $R_{ab} = R_{Th} = 9\Omega$

by the Mesh Analysis $i_2 = -2A$

@ 1st mesh

$-12 + 6i_1 + 12(i_1 - i_2) = 0$

$i_1 = -2/3\,A$

KVL @ blue direction

$-12 + V_1 + V_2 + V_{ab} = 0$

$-12 + (6i_1) + 3i_2 + V_{ab} = 0$

$V_{ab} = 22V = V_{Th}$

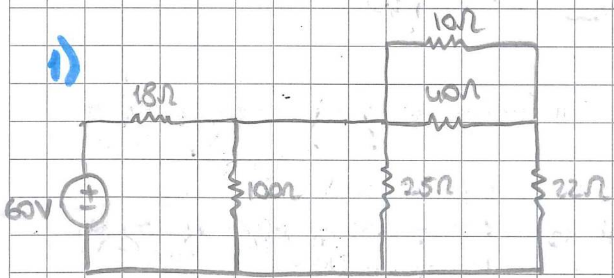
$P_{max} = \dfrac{V_{Th}^2}{4R_{Th}}$

$P_{max} = \dfrac{22^2}{4 \cdot 9}$

$= 13{,}44W$

* PROBLEM SOLVING PART 1 *

1)



Look at the figure and,

a) Use voltage division to find the voltage drop across the 18Ω resistor positive at the left.

$R_{eq} = 100\Omega \| 25\Omega \| 22\Omega + \dfrac{(10\Omega \| 40\Omega)}{8}$

$R_{eq} = 12\Omega$ $\qquad V_{18\Omega} = 60 \cdot \dfrac{18}{18+12} = 36V$

b) find the current flowing in the 18Ω with using your result in (a).

$I_{18\Omega} = \dfrac{36V}{18\Omega} = 2A$

c) Starting with your result in (b) and use current division to find $I_{25\Omega}$.

$I_{25\Omega} = (2A) \cdot \dfrac{100\Omega \| 25\Omega \| 30\Omega}{25\Omega} = 0{,}96A$

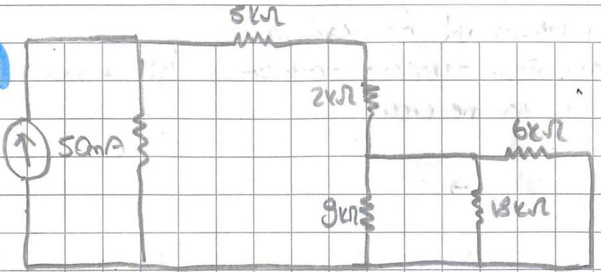
d) Using your result in (c), find $V_{25\Omega}$

$V_{25\Omega} = (0{,}96A)(25\Omega) = 24V$

e) find $V_{10\Omega}$ with voltage division

$V_{10\Omega} = (24) \dfrac{8}{22 + (10 \| 40)} = 6{,}4V$

2)



find the results according to figure

a) $I_{10k\Omega}=?$ (by current division)

$R_{eq}=5k+2k+(9k\|18k\|6k)=10k\Omega$

$I_{10k\Omega}=(50m)\frac{10k\|10k}{10k}=\underline{25mA}$

b) $V_{10k\Omega}=?$

$V_{10k\Omega}=(25mA)(10k\Omega)=\underline{250V}$

c) $V_{2k\Omega}=?$ (by voltage division)

$V_{2k\Omega}=(250)\frac{2k}{5k+2k+3k}=\underline{50V}$

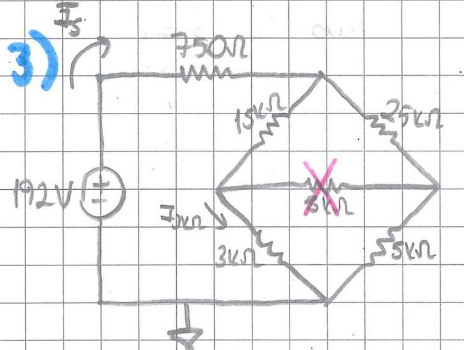
d) $I_{2k\Omega}=?$

$I_{2k\Omega}=\frac{50V}{2k\Omega}=\underline{2,5mA}$

e) $I_{18k\Omega}=?$ (by current division)

$I_{18k\Omega}=(25m)\frac{(9k\|18k\|6k)}{18k}=\underline{4,167mA}$

3)



$15k\Omega\times5k\Omega=3k\Omega\times25k\Omega$

↳ so we can cancel the 5kΩ

Find the power dissipated in the 3kΩ resistor in the circuit.

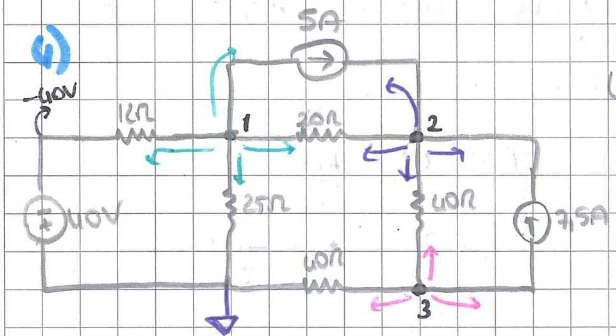
$R_{eq}=750+18k\|30k=12k\Omega$

$I_s=\frac{192V}{12k\Omega}=16mA$

$I_{3k\Omega}=(16m)\frac{18k\|30k}{18k}=10mA$

$P_{3k\Omega}=i^2R=(10mA)^2(3k\Omega)=\underline{0,3W}$

4)



Use the node voltage method to find the total power dissipated in the circuit.

@ node 1 → $\frac{V_1-(-40)}{12}+\frac{V_1}{25}+5A+\frac{V_1-V_2}{20}=0$

@ node 2 → $-7,5A+\frac{V_2-V_3}{40}+\frac{V_2-V_1}{20}-5A=20$

@ node 3 → $\frac{V_3}{40}+7,5A+\frac{V_3-V_2}{40}=0$

$V_1=-10V,\ V_2=132V,\ V_3=-84V$

$I_{40V}=\frac{-10+40}{12}=2,5A$

$P_{12\Omega}=i^2R=(2,5)^2(12)=75W$ diss.

$P_{25\Omega}=(-10V)^2/25=4W$ diss.

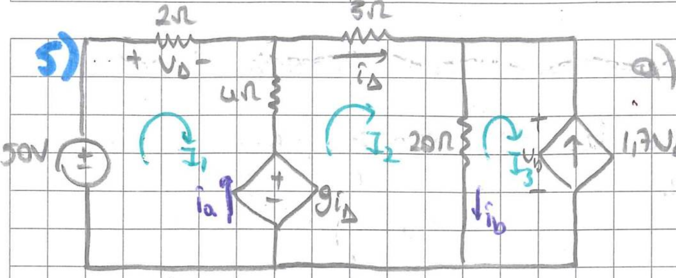
$P_{40\Omega,10W}=(-84)^2/40=176,4W$ diss

$P_{40}=(V_2-V_3)^2/R_{40}=1166,4W$ diss

$P_{20\Omega}=(V_1-V_2)^2/R_{20}=1008,2W$ diss

↳ $\underline{\Sigma P_{diss}=2430W}$

5)



a) Use the mesh current method to find which sources in the circuit are generating power.

@ 1st mesh $\rightarrow -5V + 2I_1 + 4(I_1 - I_2) + 9i_\Delta = 0$

$6I_1 - 4I_2 + 9i_\Delta = 0$

@ 2nd mesh $\rightarrow -9i_\Delta + 4(I_2 - I_1) + 5i_2 + 20(I_2 - I_3) = 0$

$-9i_\Delta + 29I_2 - 4I_1 - 20I_3 = 0$

constant equations: $i_\Delta = I_2$

$i_a = I_2 - I_1 = 21A$ $\quad i_3 = -1{,}7V_\Delta$

$i_b = I_2 - I_3 = -1A$ $\quad V_\Delta = 2I_1$

$V_b = -20V$

$$\underbrace{\qquad\qquad}$$

$I_1 = -5A,\ I_2 = 16A,\ I_3 = 17A,\ V_\Delta = -10V,\ i_\Delta = 16A$

$P_{50V} = (-)(50)(-5A) = 250W$ (absorbing)

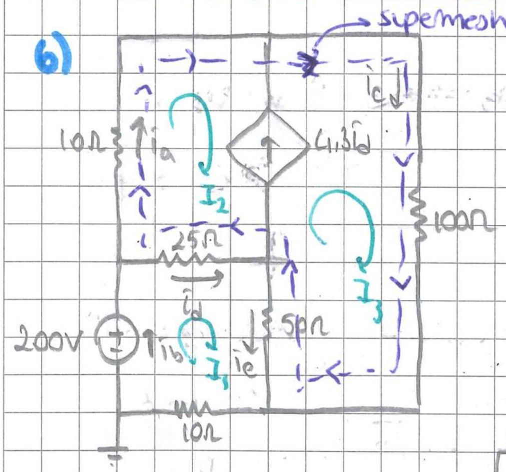
$\rightarrow P_{9i_\Delta} = (-)(144)(21A) = -3024W$ (delivering/generating)

$\rightarrow P_{1{,}7V_\Delta} = (-)(-17)(-20) = -340W$ (delivering/generating)

} These two sources are generating power in the circuit.

* PROBLEM SOLVING PART 2 *

6)



a) Use the mesh current method to find the branch currents in $i_a - i_e$ in the circuit.

@ mesh 1 $\rightarrow -200 + 85I_1 - 25I_2 - 50I_3 = 0$ ①

@ supermesh $\rightarrow 150I_3 - 75I_1 + 35I_2 = 0$ ②

constant eq: $4{,}3i_d = I_3 - I_2 \quad i_d = I_1 - I_2$

$$\underbrace{4{,}3I_1 - 3{,}3I_2 - I_3 = 0}$$ ③

$$\begin{bmatrix} 85 & -25 & -50 \\ -75 & 35 & 150 \\ 4{,}3 & -3{,}3 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 200 \\ 0 \\ 0 \end{bmatrix}$$

$I_1 = 4{,}6A$
$I_2 = 5{,}7A$
$I_3 = 0{,}97A$

$i_a = I_2 = 5{,}7A$
$i_b = I_1 = 4{,}6A$
$i_c = I_3 = 0{,}97A$
$i_d = I_1 - I_2 = -1{,}1A$
$i_e = I_1 - I_3 = 3{,}63A$

b) Show that $\sum P_{diss} = \sum P_{gen}$

$P_{200V} = -920W$
$P_{4{,}3i_d} = -399{,}68W$

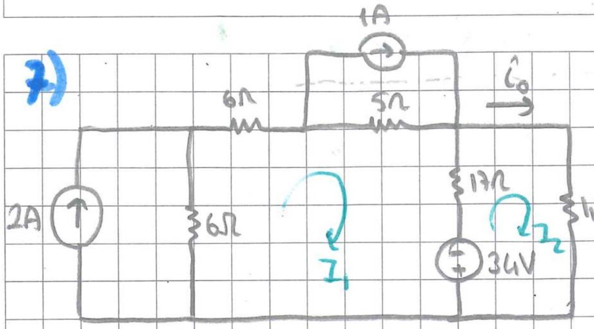
} $\sum P_{gen} = -1319{,}68W$

$P_{10\Omega} = 324{,}9W$
$P_{100\Omega} = 94{,}09W$
$P_{50\Omega} = 658{,}84W$
$P_{10\Omega} = 211{,}6W$
$P_{25\Omega} = 30{,}25W$

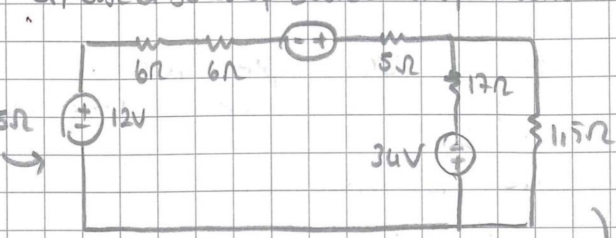
} $\sum P_{diss} = 1319{,}68W$

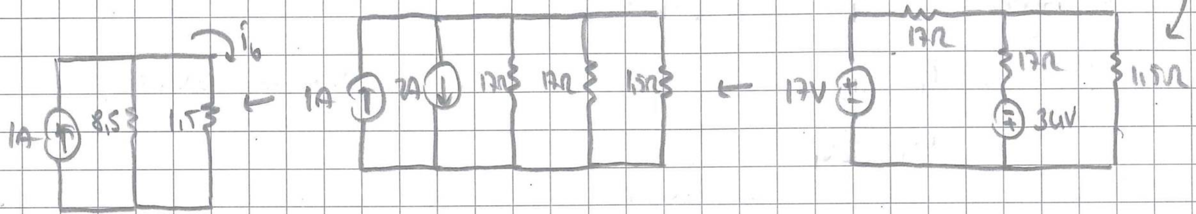
$\sum P_{diss} = \sum P_{gen}$

7)



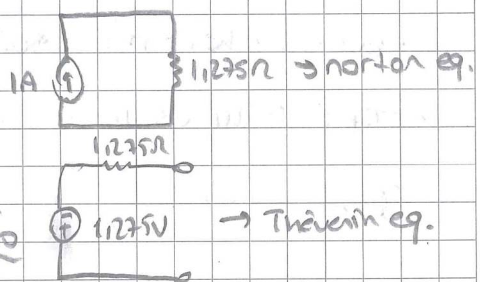
a) Use a series of source transformation to find i_o.





$$i_o = (-1)\ \frac{8{,}5 \parallel 1{,}5}{1{,}5} = \underline{-0{,}85A}$$

b) Verify your solution by using mesh analysis.



@ mesh 1 → $34I_1 - 17I_2 = 51$

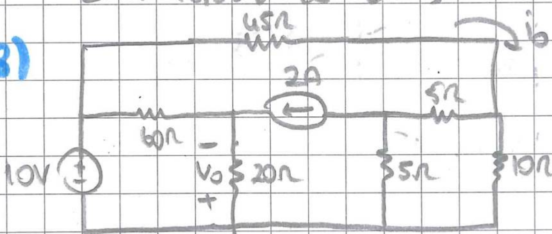
@ mesh 2 → $-17I_1 + 16{,}5I_2 = -34$

$I_1 = 1{,}07A$

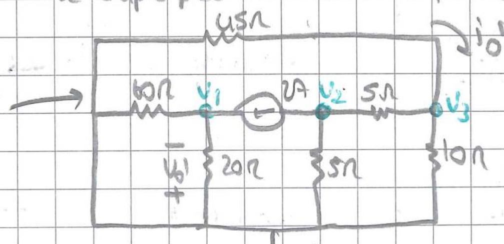
$I_2 = -0{,}85A = i_o$

$$\begin{bmatrix} 34 & -17 \\ -17 & 16{,}5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 51 \\ -34 \end{bmatrix}$$

8)



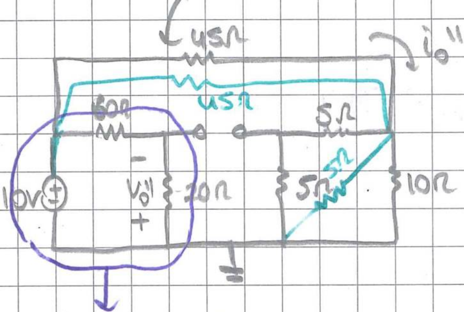
Use superposition to solve for i_o and V_o in the circuit.



@ V_2 node

$$\frac{V_2}{5} + 2A + \frac{V_2 - V_3}{5} = 0$$

$$2V_2 - V_3 = -10$$



$$\begin{bmatrix} 2 & -1 \\ -18 & 29 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$$

@ V_3 node

$$\frac{V_3}{10} + \frac{V_3}{45} + \frac{V_3 - V_2}{5} = 0$$

$$29V_3 - 18V_2 = 0$$

$V_2 = -7{,}25V$

$V_3 = -4{,}5V$

$$V_o'' = (-10)\ \frac{20}{20+60} = -2{,}5V$$

$$i_o' = -V_3/45 = 0{,}1A$$

$$i_o'' = \frac{10V}{45+5} = 0{,}2A$$

$$V_o' = \underbrace{2 \cdot \frac{20 \parallel 60}{20}}_{(1{,}5A)} \cdot 20 = -30V$$

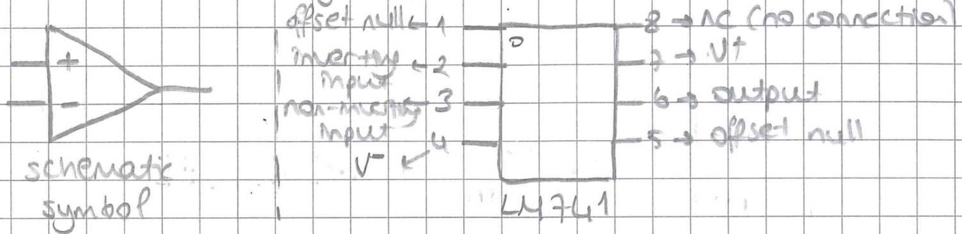
$$i_o = i_o' + i_o'' = 0{,}3A$$

$$\underline{V_o = V_o' + V_o'' = -32{,}5V}$$

* OPERATIONAL AMPLIFIERS (OP-Amps) *

An Op-amps is a DC-coupled high-gain electronic voltage amplifier with a differential input and usually a single-ended output

→ In biomedical systems, we use instrumentation amplifiers.



schematic symbol

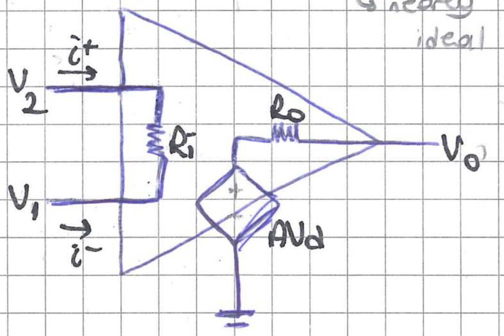
* if we want to amplifier voltage, we need to give energy.

* even we have no input, there will be an output.

* offset nulls use for finding output for no input, we should connect potentiometers.

* Op-Amps in Real Life *

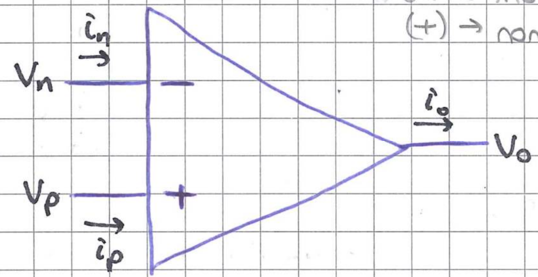
↳ nearly ideal



R_i → input resistor (very large)
R_o → output resistor (very small)
A → gain
V_d → voltage difference $V_d = V_2 - V_1$
V_o → output voltage

* Ideal Op-Amps *

(−) → inverting
(+) → non-inverting



→ $V^+ = V^-$ $\quad V_n \approx V_p$

→ $i_n = i_p \approx 0$ $\quad (R_i = \infty)$

} we will assumed that

$A_V = \dfrac{V_o}{V_p - V_n}$

* if we have no feedback resistor → open voltage gain

We use dB (decibels) → $10 \log (P/P_{ref})$, or $20 \log (V/V_{ref}) = X$ dB

Ex: $20 \log (V_o/V_{in}) = 20 \log (A) = 100$ dB

$A = 10^5$

! A is frequency dependent !

	voltage range	output voltage
linear region →	$V^- < AV_d < V^+$	$V_o = AV_d$
positive saturation →	$AV_d > V^+$	$V_o \sim V^+$
negative saturation →	$AV_d < V^-$	$V_o \sim V^-$

*** Common Mode Rejection (rejects noises) ***
(CMRR)

$$A_{CM} = \left| \frac{V_{OCM}}{V_{CM}} \right| \qquad CMRR = \left| \frac{A}{A_{CM}} \right| \qquad CMRR_{dB} = 20\log \left| \frac{A}{A_{CM}} \right| dB$$

→ A is the differential gain, A_{CM} is the common mode gain

! if slew rate ↑, CMRR sensibility ↓

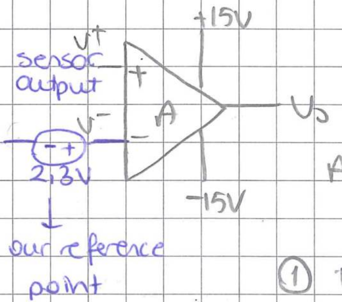
! if we choose $R_i = \infty$, $A = \infty$, $R_o = 0$ in real Op-Amps, we get ideal Op-Amps.

Ex: We want to build a sensor;

Temperature sensor → voltage

T(C°)	V(volt)
37.0	2,05
37.1	2,1
37.2	2,15
37.3	2,2
37.4	2,25
37.5	2,3

If we have 37.5°C and higher than this, we want to call doctor. and take $A = 10^5$.



$A(V^+ - V^-) = V_o$
↳ 2,3V our critical point

① $V_o = (V_{so} - 2{,}3)10^5$

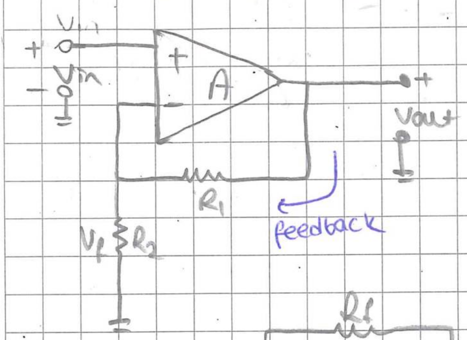
$= (2{,}29 - 2{,}3)10^5 = -1000V$ → negative saturates for all sensor outputs as; $V_{so} < 2{,}3$

② $V_o = (2{,}301 - 2{,}3)10^5$

$= +100V$ → positive saturates for all sensor outputs as; $V_{so} > 2{,}3$

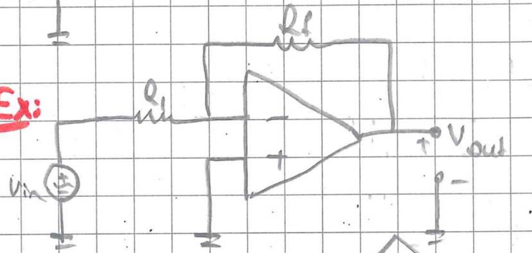
↳ This types of circuits are called voltage comparator circuit.

★ Feedback Mechanism (closed loop Gain) ★



$$G = \frac{V_{out}}{V_{in}} = 1 + \frac{R_1}{R_2}$$

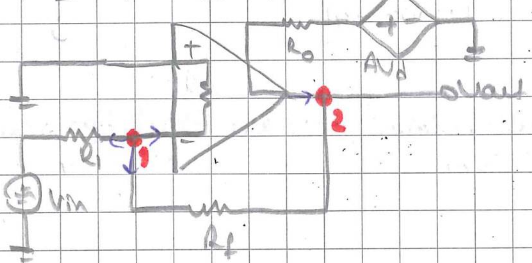
Ex:



For a 741 Op-Amp ($A = 200{,}000$, $R_i = 2M\Omega$, $R_o = 75\Omega$), $V_{in} = 5\sin 3t\ mV$, $V_{out} = ?$

$R_1 = 4.7k\Omega$

$R_f = 47k\Omega$



@ node 1

$$0 = \frac{-V_d - V_{in}}{R_1} + \frac{-V_d - V_{out}}{R_f} + \frac{-V_d}{R_i}$$

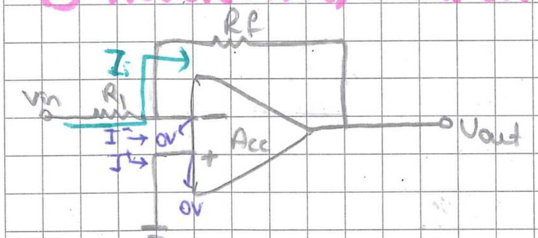
@ node 2

$$0 = \frac{V_{out} + V_d}{R_f} + \frac{V_{out} - AV_d}{R_o}$$

$$V_{out} = \left[\frac{R_o + R_f}{R_o - AR_f}\left(\frac{1}{R_1} + \frac{1}{R_f} + \frac{1}{R_i}\right) - \frac{1}{R_f}\right]^{-1} \frac{V_{in}}{R_1}$$

$$= -9.99\, V_{in} = \underline{\underline{-49.99 \sin 3t\ mV}}$$

① INVERTING AMPLIFIER

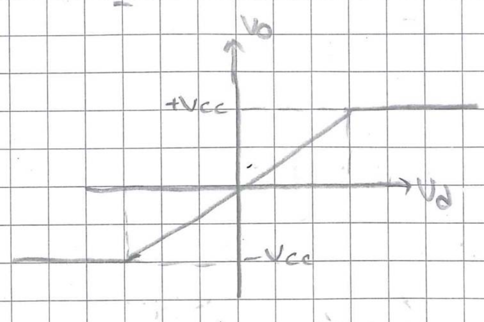


$$\boxed{V_{out} = V_{in}\left(-\frac{R_f}{R_1}\right)}$$

$$\frac{V_i - 0}{R_1} = \frac{0 - V_{out}}{R_f}$$

$V^+ = V^-$

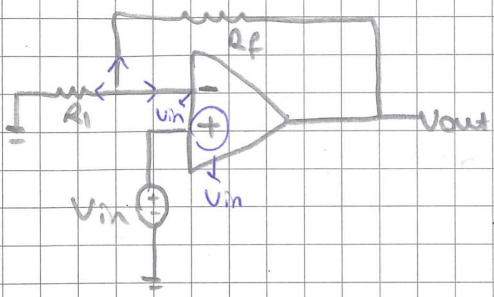
$I^+ = I^- = 0$



→ V_{cc}^+ / V_{cc}^- are the supplying voltages for work the Op-Amp.

→ There would be a phase difference with 180°C.

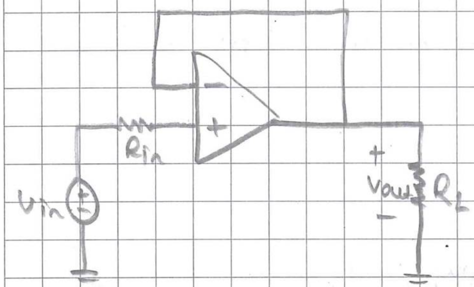
② NON-INVERTING AMPLIFIER



$$\boxed{V_{out} = \left(\frac{R_f}{R_1} + 1\right) V_{in}}$$

$$\frac{V_{in} - 0}{R_1} + \frac{V_{in} - V_0}{R_f} = 0$$

③ VOLTAGE FOLLOWER (unity gain amplifier)

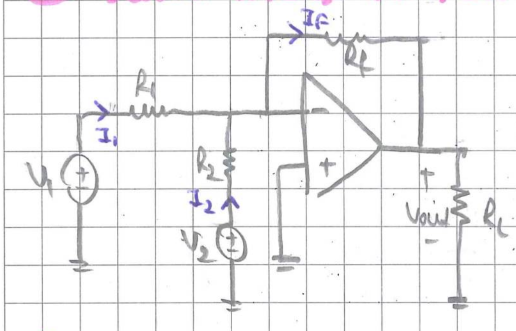


$R_1 \to \infty$, $R_f \to 0$, $\boxed{V_{out} = V_{in}}$

→ The input impedance of the Op-amp is very high, giving effective isolation of the output from the signal source.

Modulation → $\underbrace{M(t)}_{\text{message signal}} + \underbrace{C(t)}_{\text{carrier signal}} = S(t)$ | we use in filters

④ SUMMING AMPLIFIER

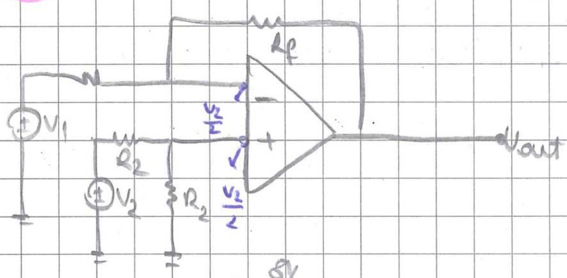


$$I_1 + I_2 = I_f$$

$$\boxed{V_{out} = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_1} V_2\right)}$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{0 - V_0}{R_f}$$

⑤ DIFFERENCE AMPLIFIER

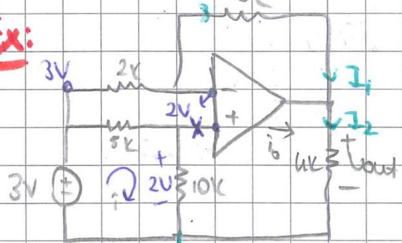


$$\boxed{V_{out} = V_2 - V_1}$$

$$\frac{V_1 - \frac{V_2}{2}}{R_1} = \frac{\frac{V_2}{2} - V_0}{R_1}$$

$$V^+ = \frac{V_2 R_2}{2R_2} \Rightarrow V^+ = V^- = \frac{V_2}{2}$$

Ex:



$-3V + 5i + 10i = 0$

$i = 0{,}2$

$V^+ = V_{10k\Omega} = 2V$

Find the i_0, V_{out}.

$V^+ = V^- = 2V$

$$\frac{3V - 2V}{2k} = \frac{2V - V_{out}}{8k}$$

$\underline{V_{out} = -2V}$

KCL → $i_0 + I_1 = I_2$

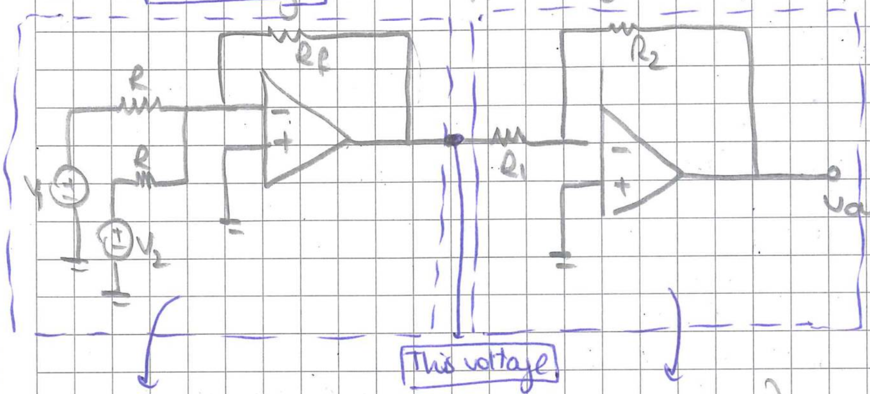
$$i_0 = \frac{-2V}{4k} - \frac{2V - (-2V)}{8k}$$

$\underline{i_0 = -1mA}$

★ Cascading Op-Amps ★

Op-Amps can be combined in stages to create the desired relationship between the outputs and inputs.

→ $\boxed{\text{The voltage}}$ is not affected by the circuit on the right.



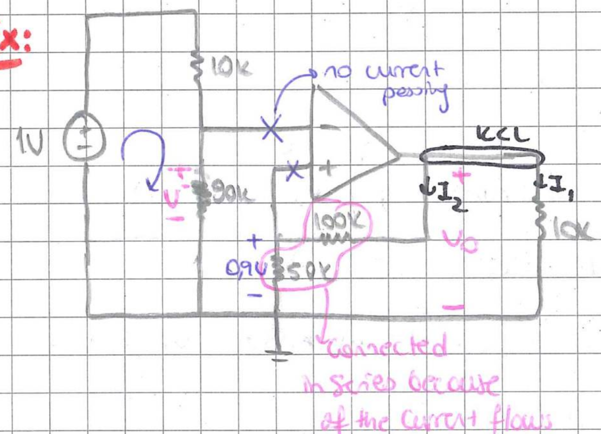
$$V_{out} = \frac{R_2 R_f}{R_1 R}(V_1 + V_2)$$

$V_{out} = \frac{R_f}{R}(V_1 + V_2)$ (summing)

$V_{out} = \frac{-R_2}{R_1} V_{in}$ (inverting)

We can think like that we multiply two gains.

Ex:



Find the I_0 and V_0.

→ we need to find $V^+ = V^-$ → KCL @ node

$V^- = 1V \cdot \frac{90k}{100k} = 0{,}9V$

$0{,}9V = V_0 \cdot \frac{50k}{150k}$

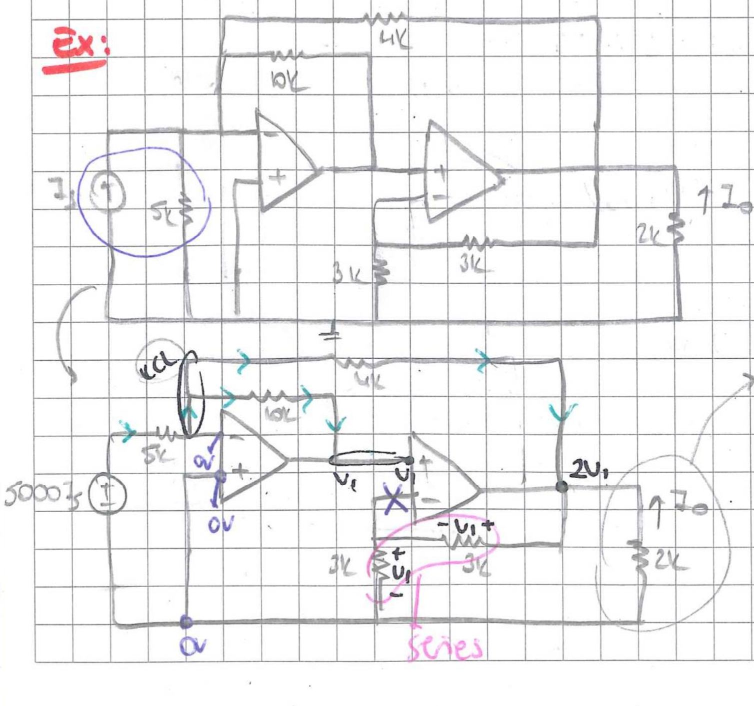
$\boxed{V_0 = 2{,}7V}$

$I_0 = I_1 + I_2$

$= \frac{2{,}7V}{10k} + \frac{2{,}7V}{150k}$

$\boxed{I_0 = 0{,}28mA}$

Ex:



Find the current gain I_0 / I_s.

KCL @ node

$\frac{5000 I_s - 0}{5k} = \frac{0 - V_1}{10k} + \frac{0 - 2V_1}{4k}$

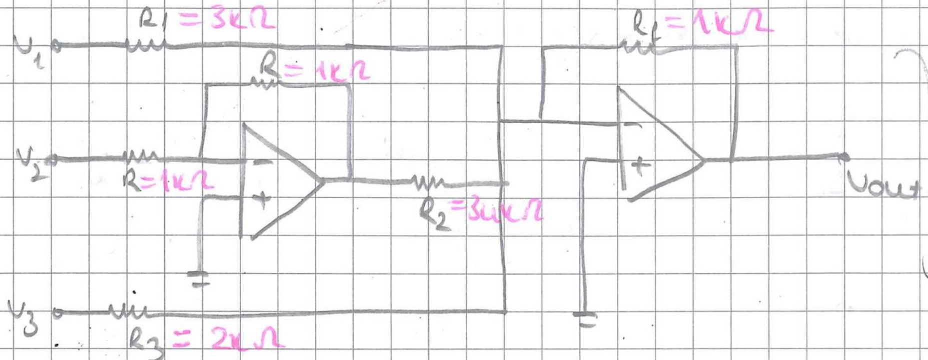
$I_s = \frac{-12V_1}{20000}$

$I_0 = \frac{0 - 2V_1}{2k} \rightarrow I_0 = \frac{-2V_1}{2000}$

$\frac{I_0}{I_s} = \frac{-2V_1/2000}{-12V_1/20000} = \frac{5}{3}$

Ex: Using only 2 op-amps, design a circuit to solve;

$$-V_{out} = \frac{V_1 - V_2}{3} + \frac{V_3}{2} \Rightarrow -V_{out} = \frac{V_1}{3} - \frac{V_2}{3} + \frac{V_3}{2}$$



cascaded circuits

benzer so,

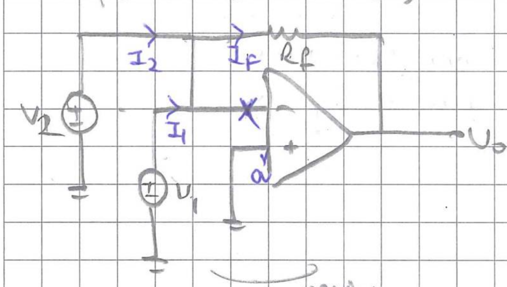
$R_1 = 3k\Omega$

$R_2 = 3k\Omega$

$R_3 = 2k\Omega$

$R = R = R_f = 1k\Omega$

for solve ↑ this;

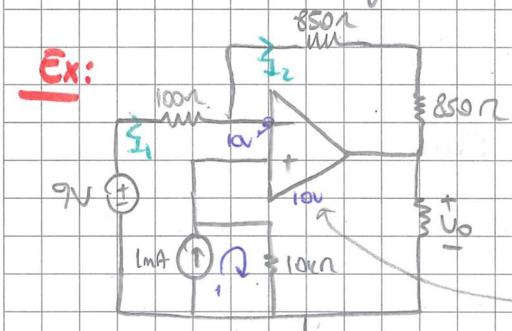


$$I_1 = I_2 = I_f$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = \frac{-V_o}{R_f}$$

$$-V_o = \frac{R_f V_1}{R_1} + \frac{R_f V_2}{2}$$

Ex:



Determine the value of V_o.

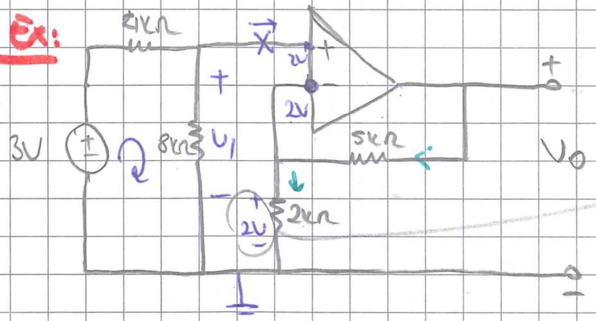
$V^- = V^+$, $I^- = I^+ = 0$

$V^+ = 1mA \times 10k\Omega = 10V$ → come from loop 1

$V^+ = V^- = 10V$

$$I_1 = I_2 \rightarrow \frac{9V - 10V}{100\Omega} = \frac{10V - V_o}{1700\Omega} \Rightarrow \underline{V_o = 27V}$$

Ex:



Calculate the V_o.

$V^+ = V^-$

$$V_1 = (3V) \cdot \frac{8k\Omega}{12k\Omega} = 2V$$

$I^- = I^+ = 0$

$$2V = V_o \cdot \frac{2k\Omega}{7k\Omega} \rightarrow \underline{V_o = 7V}$$

* pico 10^{-12}
nano 10^{-9} (n)
micro 10^{-6} (μ)

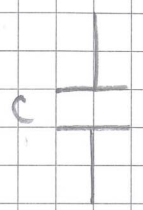
* ENERGY STORAGE ELEMENTS *

***CAPACITORS:** A capacitors is a linear circuit element which stores energy in the electric field in the space between two conducting bodies occupied by a material with permittivity ε.

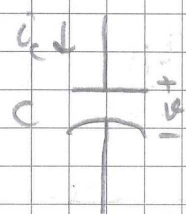
$C = \varepsilon \cdot \frac{A}{d}$ $\qquad C = \frac{Q}{V}$

$\varepsilon = \varepsilon_r \cdot \varepsilon_0 \rightarrow$ vacuum permittivity

ε_r ↓ relative dielectric constant



not polarized



polarized

$$\boxed{v_c(t) = \frac{1}{C}\left[\int_{t_0}^{t} i(t)\,dt + V(t_0)\right]}$$

$$\boxed{i_c(t) = C \cdot \frac{dV_c(t)}{dt}}$$

$$\boxed{P = CV\frac{dV}{dt}}$$

$$\boxed{W = \frac{1}{2}CV^2}$$

charged: by applying current to its terminals

discharged: when it provides current from its terminals.

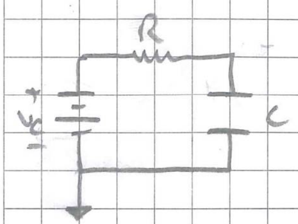
* When we give on the capacitor

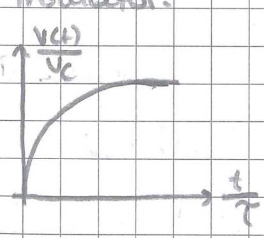
$\rightarrow$ DC voltage : 0 current (blocked) $\rightarrow$ acts like a open circuit.

$\rightarrow$ AC voltage : $i_c(t) = C \cdot \frac{dV_c(t)}{dt}$

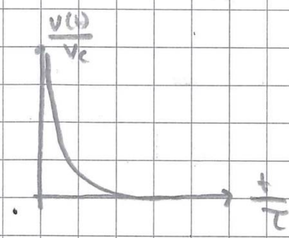
current "leads" voltage, voltage "lags" current.

*Real capacitor: does dissipate energy due to leakage of charge through its insulator.





charging



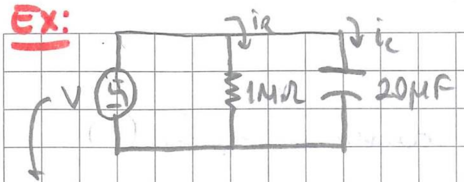
discharging

$\tau = RC$
↳ time constant

$V(t) = V_c(1 - e^{-t/\tau})$

$V(t) = V_c e^{-t/\tau}$

Ex:



$100\sin 2\pi t$

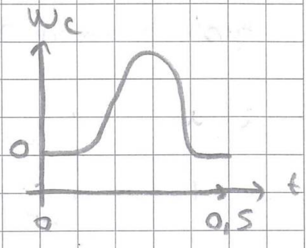
Find the max energy stored in capacitor, at $0<t<0{,}5$

$2\pi f \rightarrow 2\cdot 0{,}5 = 1 \quad f=1$

$w_c = \frac{1}{2} C V^2$

$= \frac{1}{2}(20\mu F)((100\sin 2\pi t)V)^2$

$= 100\cdot \sin^2(2\pi t)\ mJ$



$i_c = C\cdot\frac{dV}{dt}$

$= 20\mu F \frac{d}{dt}(100\sin(2\pi t))$

$= 4\pi\cos(2\pi t)\ mA$

$i_R = \frac{V}{R} = \frac{100\sin(2\pi t)}{10^6} = 0{,}1\sin(2\pi t)\ mA$

* the current of capacitors and voltage of capacitors has $90°$ of phase shift.

* source current and capacitor current has $180°$ of phase shift.

→ Capacitors in Parallel: $i_{in} = i_1 + i_2 + i_3 - -$ $\quad C_{eq} = C_1 + C_2 + C_3 - - - = \sum_{p=1}^{P} C_p$

$i_{in} = C_{eq}\frac{dV}{dt}$ $\quad V = V_1 = V_2 = V_3 - -$

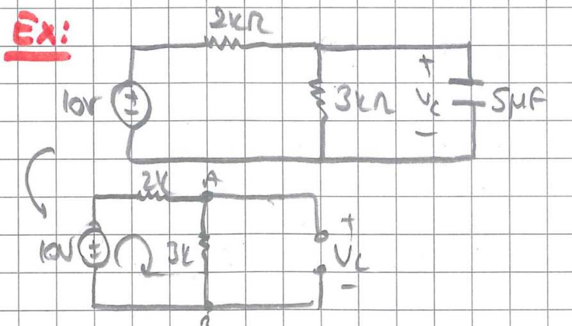
→ Capacitors in Series: $V_{in} = V_1 + V_2 + V_3 + - - -$ $\quad C_{eq} = \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots\right]^{-1} = \left[\sum_{s=1}^{S}\frac{1}{C_s}\right]^{-1}$

$V_{in} = \frac{1}{C_{eq}}\int_{t_0}^{t} i\,dt$

* The voltage across a capacitor must be a continuous function; the current flowing across a capacitor can be discontinuous.

** An ideal capacitor act like an open circuits when a DC voltage or current has been applied for at least 5τ.

Ex:

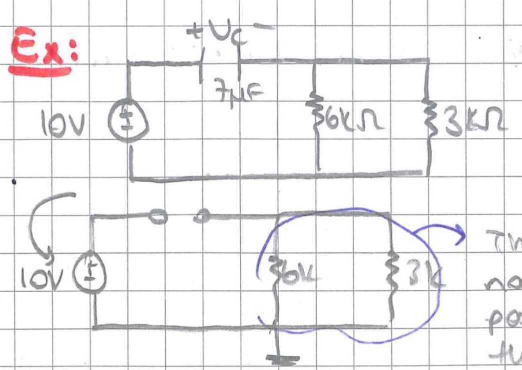


$V_c = ?$

$V_{AB} = 10\cdot\frac{3}{5} = 6V = V_c$

$\underline{\underline{V_c = 6V}}$

Ex:

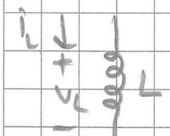


$V_c = ?$

There is no current passing through this part.

$\underline{\underline{V_c = 0V}}$

*** INDUCTORS:** a component in an electric or electronic circuit which possesses inductance, also called a coil, choke or reactor is a passive two-terminal electrical component



$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)}$$

$$\boxed{V_L = L\,\frac{di(t)}{dt}}$$

$$L = \mu\,\frac{N^2 A}{\ell}$$

$$\boxed{W = \frac{1}{2} L i^2}$$

* if we have decrease current → (−) voltage
constant current → 0 voltage
increase current → (+) voltage

* when we give on an inductor
→ DC voltage : acts like a short circuit
→ AC voltage : $i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt$

voltage "leads" current, current "lags" voltage.

* current must be continous.

→ Inductors in Parallel: $i_n = i_1 + i_2 + i_3 + \dots$

$$i_n = \frac{1}{L_{eq}}\int_{t_0}^{t} v(t)\,dt$$

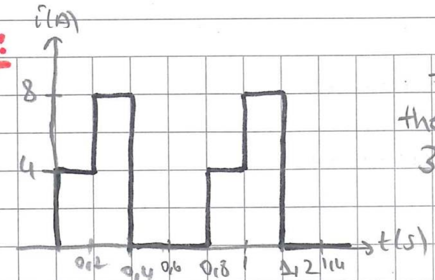
$$L_{eq} = \left[\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \dots\right]^{-1} = \left[\sum_{p=1}^{n} \frac{1}{L_p}\right]^{-1}$$

→ Inductors in Series: $V_{in} = V_1 + V_2 + V_3 + \dots$

$$V_{in} = L_{eq}\,\frac{di(t)}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + \dots = \sum_{s=1}^{s} L_n$$

Ex:



The current flowing through 33 mF capacitor is shown. Assume the passive sign convention. Compute the voltage at 300ms, 600ms and 1.1s.

$V_c(t) = \frac{1}{C}\int I_c(t)dt + V_c(t_0)$

$V_c(0{,}2s) = \frac{4 \cdot 0{,}2}{33mF} = 24{,}2F$

$V_c(0{,}4s) = \frac{8 \cdot 0{,}2}{33mF} + 24{,}2 = 72{,}7V$

$V_c(0{,}3s) = \frac{1}{2}\left[V(0{,}2) + V(0{,}4)\right] = \underline{\underline{48{,}5V}}$

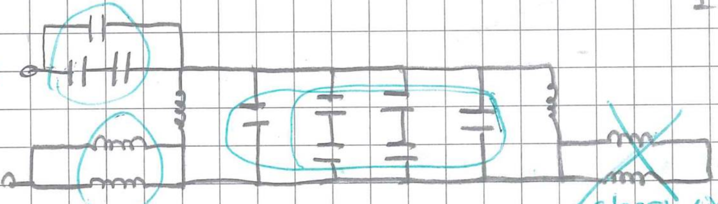
$V_c(0{,}6) = \text{constant @ } 72{,}7V = \underline{\underline{72{,}7V}}$

$V_c(1s) = \frac{4 \cdot 0{,}2}{33mF} + 72{,}7V = 97V$

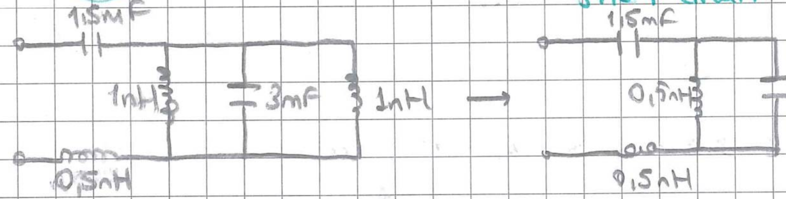
$V_c(1{,}2) = \frac{8 \cdot 0{,}2}{33mF} + 97V = 145{,}5V$

$V_c(1{,}1s) = \frac{1}{2}\left[V(1s) + V(1{,}2)\right] = \underline{\underline{121{,}2V}}$

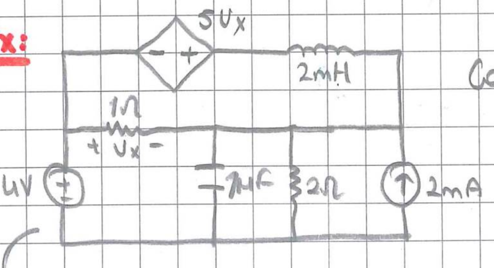
Ex:



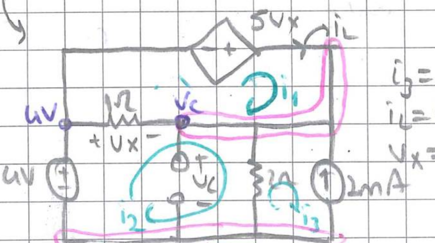
Reduce the network to the smallest possible number of components if each inductor is 1nH and capacitor is 1mF.



Ex:



Calculate the energy stored in each energy storage element.



$i_3 = -2mA$

$i_L = i_1$

$V_x = i_2 - i_1$

@ 1st mesh

$-5V_x + 1(i_1 - i_2) = 0$

$6i_1 - 6i_2 = 0$

$i_1 = i_2$

@ 2nd mesh

$-4 + V_x + 2(i_2 - i_3) = 0$

$-i_1 + 3i_2 = 4 - \underbrace{4 \cdot 10^{-3}}_{\text{can be negligible}}$

$i_1 \cong 2A,\ i_2 \cong 2A$

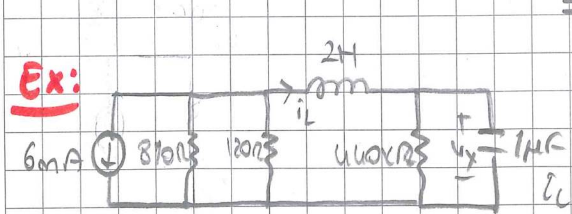
$V_c = 2\Omega \times 2A = 4V$

→ capacitor → open c.
inductor → short c.

$W_c = \frac{C V_c^2}{2} = \frac{(1\mu)(4V)^2}{2} = \underline{\underline{8mJ}}$

$W_L = \frac{L \cdot i_L^2}{2} = \frac{(2m)(2A)^2}{2} = \underline{\underline{4mJ}}$

Ex:



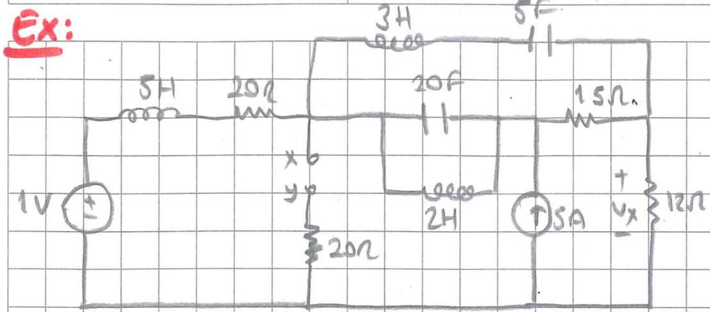
$V_x = ?$

$i_L = -6mA \frac{104{,}2\Omega}{440k\Omega} = -1{,}42\mu A$

$R_{eq} = \left[\frac{1}{810} + \frac{1}{120} + \frac{1}{440000}\right]^{-1} = 104{,}2\Omega$

$V_x = i_L \times 440k\Omega = \underline{\underline{-624{,}8mV}}$

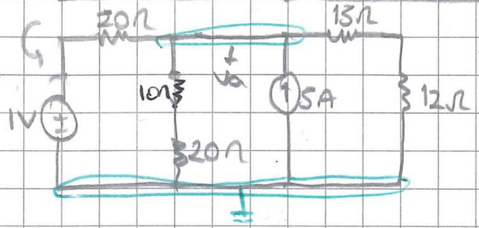
Ex:



a) x-y → 10Ω and determine V_x

b) x-y → 1H and determine V_x

c) x-y → 1F and determine V_x



KCL @ Va;

$$\frac{1-V_a}{20} + 5 - \frac{V_a}{30} - \frac{V_a}{27} = 0$$

$V_a = 41{,}7V$

$$a \rightarrow V_x = \frac{V_a}{27} 12 = 18{,}53V$$

$$b \rightarrow \frac{1-V_a}{20} + 5 - \frac{V_a}{20} - \frac{V_a}{27} = 0$$

$V_a = 36{,}85V \qquad V_x = \frac{V_a}{27} 12 = 16{,}4V$

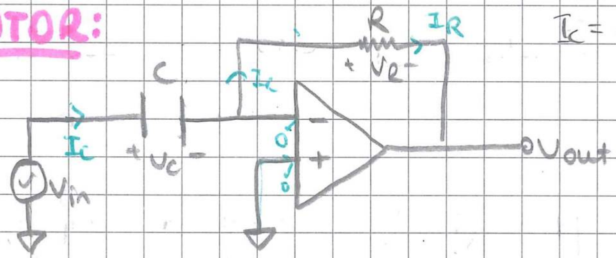
$$c \rightarrow -5 + \frac{V_a}{27} + \frac{(V_a - 1)}{20} = 0$$

$V_a = 58{,}02V$

$V_x = 25{,}8V$

1st ORDER Op-AMPs CIRCUITS

DIFFERENTIATOR:

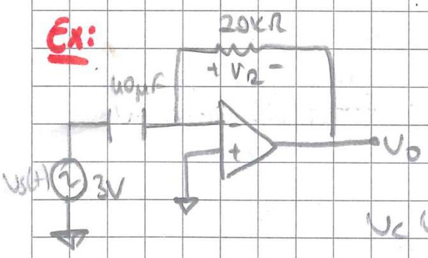


$$I_c = C\frac{dV_{in}}{dt} = \frac{0 - V_{out}}{R}$$

$\tau \rightarrow RC$

$$V_{out} = -\tau \frac{dV_{in}}{dt}$$

Ex:



Suppose that $V_s(t) = 3V\, u(t - 5s)$.

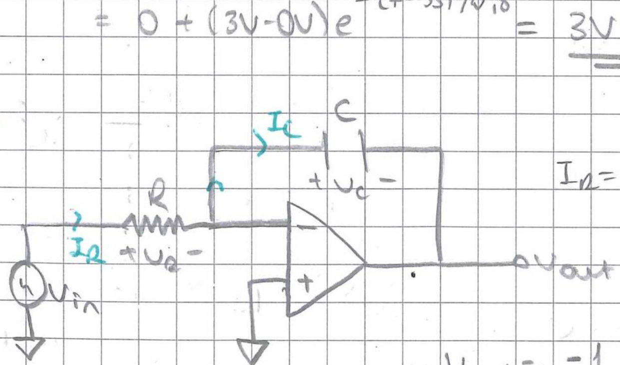
→ initial condition of $V_c = 0V$ when $t < 5s$,

→ final condition of $V_c = 3V$ when $t > 5\tau$,

$$V_c(t) = V_{c_{in}} + (V_{c_{final}} - V_{c_{in}}) e^{-(t-t_0)/\tau} \qquad \tau = RC = 0{,}8$$

$$= 0 + (3V - 0V) e^{-(t-5s)/0{,}8} = 3V e^{-(t-5s)/0{,}8} \text{ when } t > 5s$$

INTEGRATOR:



$$I_R = \frac{V_{in}}{R} \qquad I_c = C\frac{dV_{in}}{dt}$$

$$V_{out} = \frac{-1}{RC} \int_{t_1}^{t_2} V_{in}(t)\,dt + V_0(t)$$

! differentiator → if our input voltage is sinusoid, output voltage lags input voltage.

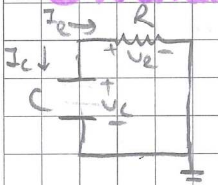
integrator → if our input voltage is sinusoid, output voltage leads input voltage.

the output must be continuous.

* RC AND RL CIRCUITS *

* RC CIRCUITS *

① Natural Response:



$V_C = V_R$ $\quad V_C(t) = V_0 e^{-t/\tau}$ $\quad P_R(t) = \frac{V_0^2}{R} e^{-2t/\tau}$

$I_R(t) = -I_C(t) = \frac{V_0}{R} e^{-t/\tau}$ $\quad W_C(t) = \frac{C V_0^2}{2}\left[1 - e^{-2t/\tau}\right]$

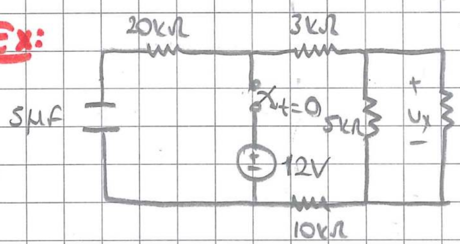
$\left.\begin{aligned} V_C &= A e^{st} \\ V_R &= \frac{1}{RC} A e^{st} \end{aligned}\right\} s = \frac{-1}{RC}$

* @ $t=0$, the initial voltage $V(0) = V_0$ across the capacitor, capacitor acts like an open circuit.

* Capacitor stored energy at $t=0s$, released energy at $t>0s$.

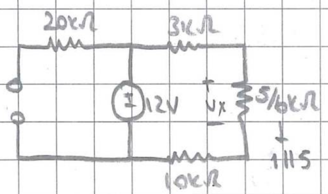
* time constant $\tau = RC$

Ex:

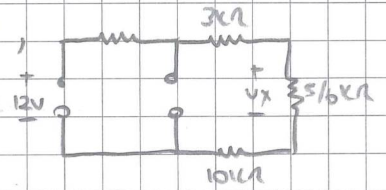


The switch above the 12V source in the circuit has been closed since just after the wheel was invented. It is finally thrown open at $t=0$.
a) compute the circuit's time constant.
b) Obtain an expression for $v(t)$ for $t>0$, calculate the energy stored in the capacitor 170ms after switch opened.

$t<0$,



$t>0$,



$V_C(0^-) = 12V$

$V_x(0^-) = 12 \cdot \frac{5/6 k\Omega}{(\frac{5}{6}+3+10)k\Omega} = 723mV$

final condition → $V_C(\infty) = 0$
$V_x(\infty) = 0$

a) $\tau = R_{eq} \cdot C = (33{,}8k\Omega)(5\mu F)$
$= \underline{169ms}$

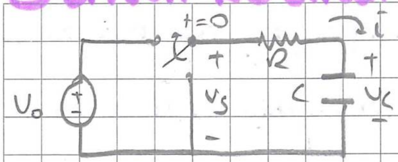
$V_x(0^+) = 12 \cdot \frac{5/6 k\Omega}{(\frac{5}{6}+3+10+20)k\Omega} = 296mV$

b) $V_x(t) = 296 e^{-5{,}9t}$ mV

$V_C(t) = 12 e^{-5{,}9t}$ V

$W_C = \frac{1}{2} C V^2 = \underline{360 e^{-11{,}8t} \mu J}$

② Driven RC Circuits:



$V_s=0,\ V_c(0^-)=0$ → Just before $t=0$

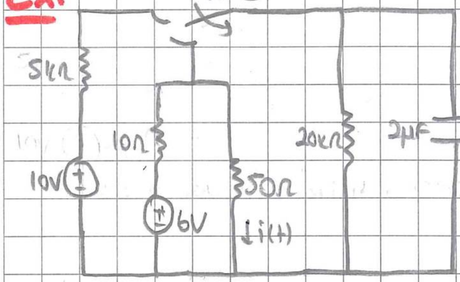
$V_s=V_0,\ V_c(0^+)=0$ → Just after $t=0$

* capacitor acts like a open circuit → $V_c(\infty)=V_0$

* $V_c(t)=V_c(\infty)+[V_c(0)-V_c(\infty)]e^{-t/\tau}$ → $V_c(t)=V_0(1-e^{-t/\tau})$

* $I_c(t)=\frac{C}{\tau}[V_c(\infty)-V_c(0)]e^{-t/\tau}$ $\tau=RC$

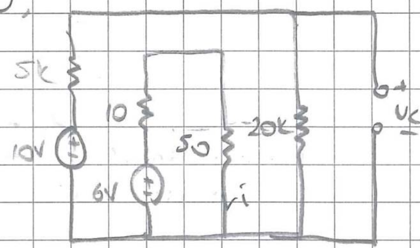
Ex:



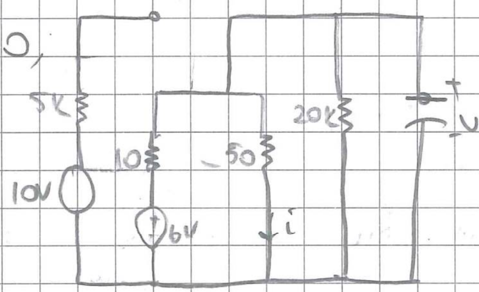
a) Obtain expression for $i(t)$ and $V_c(t)$ valid for all values of t.

b) Determine the energy remaining the capacitor at $t=33\,\mu s$.

$t<0$,



$t>0$,



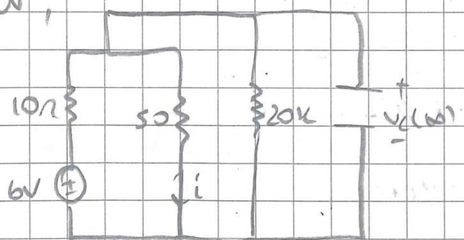
$V_c(0^-)=10\cdot\frac{20k}{5k+20k}=8V$ $*V_c(0^+)=V_c(0^-)$ $i(0^+)=\frac{8}{50}=160mA$

$i(0^-)=\frac{6}{(10+50)k}=100mA$

a) $R_{eq}=10\|50\|20k=8{,}3\Omega$

$\tau=RC=16{,}7\mu s$

$t\to\infty$,



$V_c(t)=5+(8-5)e^{-t/16{,}7\mu s}=\underline{5+3e^{-60{,}000t}}$ V

$i(t)=100+(160-100)e^{-t/16{,}7\mu s}=\underline{100+60e^{-60{,}000t}}$ mA   } $t>0$

$V_c(\infty)=6\cdot\frac{50\|20k}{10+50\|20k}=5V$

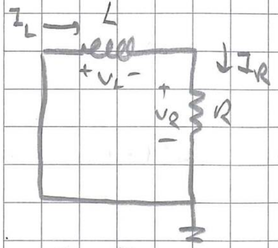
$i(\infty)=\frac{6}{60}=100mA$

b) $V_c(33\mu s)=5+3e^{-60000(33\cdot10^{-6})}=5{,}41V$

$W_c=\frac{1}{2}(2\mu)(5{,}41)^2=\underline{29{,}3\mu J}$

* RL CIRCUITS:

① Natural Response:



$$I_L(t) = I_0 e^{-t/\tau} \qquad P_R(t) = R I_0^2 e^{-2t/\tau}$$

$$V_R(t) = -V_L(t) = R I_0 e^{-t/\tau} \qquad W_R(t) = \frac{L I_0^2}{2}\left[1 - e^{-2t/\tau}\right]$$

* @ $t=0$ the initial current $i(0) = I_0$ through the inductor. Inductor acts like a short circuit

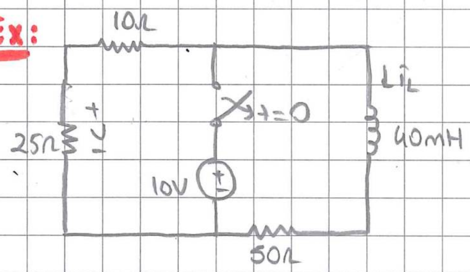
$I_L = I_R$

$I_L = Ae^{st} \rightarrow s = -\frac{R}{L}$

* Stored energy in inductor at $t=0$, released at $t>0s$.

* time constant $\tau = \frac{L}{R}$

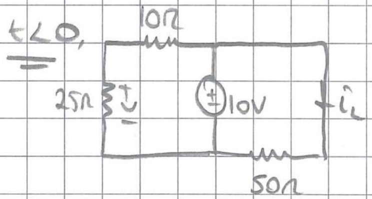
Ex:



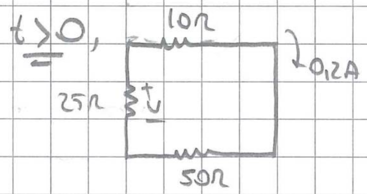
a) Obtain expressions for i_L and V which are valid for all $t \geq 0$.

b) Calculate $i_L(t)$ and $V(t)$ at $t = 470 \mu s$.

$t < 0$,



$t > 0$,



$\tau = \frac{L}{R_{eq}} = \frac{40m}{85}$ $\qquad R_{eq} = 85Ω$

$\tau = 471 \mu s$

$V(0^-) = 10 \cdot \frac{25}{35} = 7,1V$ $\qquad V(0^+) = -(25)(0,2) = -5V$

$i_L(0^-) = \frac{10}{50} = 0,2A$ $\qquad i_L(0^+) = i_L(0^-) = 0,2A$

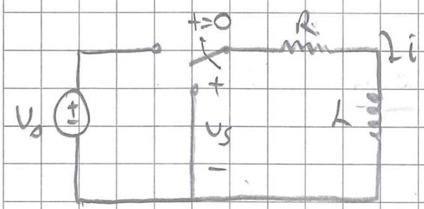
final condition $\rightarrow V(\infty) = 0,\ i_L(\infty) = 0$

a) $V(t) = -5e^{-2125t}\ V$ $\qquad$ b) at $t = 470\mu s$

$i_L(t) = 200e^{-2125t}\ mA$ $\qquad V = -5/e = -1,84V$

$i_L = 200/e = 74\ mA$

② Driven RL Circuits:



$V_s = 0,\ i(0^-) = 0 \rightarrow$ just before $t=0$

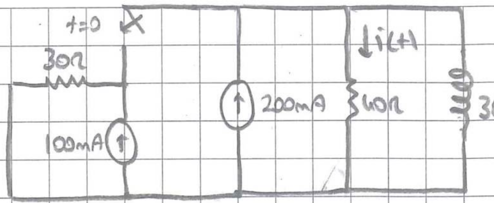
$V_s = V_0,\ i(0^+) = 0 \rightarrow$ just after $t=0$

* inductor acts like a short circuit.

* $I_L(t) = I_L(\infty) + [I_L(0) - I_L(\infty)]e^{-t/\tau}$ $\qquad \tau = \frac{L}{R}$

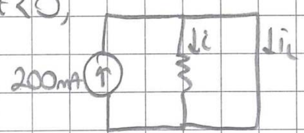
* $V_L(t) = \frac{L}{\tau}[I_L(\infty) - I_L(0)]e^{-t/\tau}$

Ex:

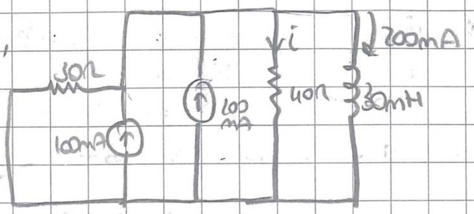


Obtain an expression for $i(t)$ as labeled in the circuit, and determine the power dissipated in the 40Ω resistor at $t = 2{,}5\,ms$.

$t<0$,



$t>0$,



$t \to \infty, \quad i(\infty) = 0$

$\tau = \frac{L}{R_{eq}} = 1{,}75\,ms$

$i(0^-) = 0$
$i_L(0^-) = 200mA$

$i_L(0^+) = i_L(0^-) = 200mA$

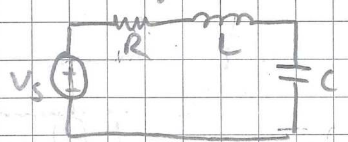
$i(0^+) = 100 \cdot \frac{1/40}{1/40 + 1/30} = 42{,}9\,mA$

$i(t) = 42{,}9\,e^{-t/1{,}75m} = 42{,}9\,e^{-571t}\ mA$

$P = i^2 R$
$= [0{,}0429\,e^{-571 \cdot 2{,}5m}]^2\,40$
$= \underline{\underline{4{,}2mW}}$

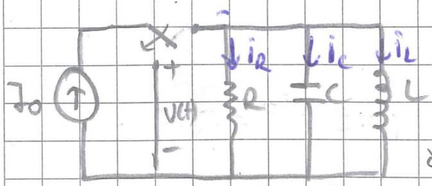
✶ RLC CIRCUITS ✶

- A second order circuit is characterized by a second-order differential equation.
- The circuit will contain at least one resistor and the equivalent of two energy storage elements.



→ We use RLC circuits in analog filters.

• Parallel RLC Circuits:



parallel RLC circuit

$-I_0 + i_R + i_L + i_C = 0$

$-I_0 + \frac{V(t)}{R} + \frac{1}{L}\int_{t_0}^{t} V(t)\,dt + C\frac{dV(t)}{dt} = 0$

derivative

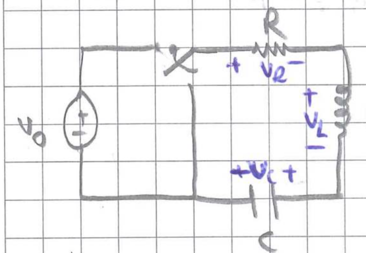
$\frac{d^2}{dt^2}V(t) + 2\alpha\frac{d}{dt}V(t) + \omega_0^2\,V(t) = 0$

α → damping coefficient

ω_0 → resonant frequency

$$\boxed{\alpha = \frac{1}{2RC}} \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

* Series RLC Circuits:



$$-V_0 + V_R(t) + V_L(t) + V_C(t) = 0$$

derivative

$$-V_0 + R\,i(t) + L\frac{d\,i(t)}{dt} + \frac{1}{C}\int_{t_0}^{t} i(t)\,dt = 0$$

$$\frac{d^2}{dt}\,i(t) + 2\alpha\frac{d}{dt}\,i(t) + \omega_0^2\, i(t) = 0$$

$$\boxed{\alpha = \frac{R}{2L}} \qquad \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

* General Solution: $x(t) = x_1 e^{s_1 t} + x_2 e^{s_2 t} + x_3$

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$x(0^+) = x_1 + x_2 + x_3 \quad x(\infty) = x_3$

$\frac{dx}{dt}(0^+) = s_1 x_1 + s_2 x_2$

* $\alpha > \omega_0 \rightarrow$ over-damped $\rightarrow x(t) = x_1 e^{s_1 t} + x_2 e^{s_2 t} + x_3$
* $\alpha = \omega_0 \rightarrow$ critically-damped $\rightarrow x(t) = e^{-\alpha t}(x_1 + x_2) + x_3$

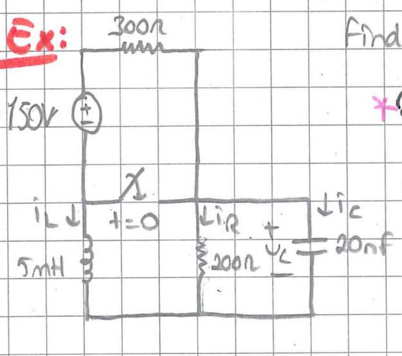
} no ossilation

* $\alpha < \omega_0 \rightarrow$ under-damped $\rightarrow x(t) = e^{-\alpha t}\left[x_1 \cos(\omega_d t) + x_2 \sin(\omega_d t)\right] + x_3$

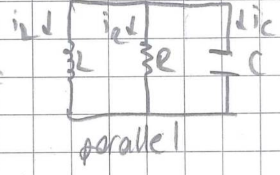
ossilation occurs

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ (neutral resonant frequency)

Ex: Find an expression for $V_C(t)$ valid for $t > 0$ in the circuit.



*(1) $t > 0$, we need to determine our RLC circuit type and find α, ω_0



$\alpha = \frac{1}{RC} = \frac{1}{2\cdot 200\cdot 20\cdot 10^{-9}} = 0{,}125\cdot 10^{6}\,s^{-1}$

$\omega_0 = \frac{1}{\sqrt{LC}} = 10^6\,rad\,s^{-1}$

} $\alpha > \omega_0$ overdamped and no ossilation.

*(2) We need to obtain $t = \infty$

$V_C(t) = V_1 e^{s_1 t} + V_2 e^{s_2 t} + V_3$

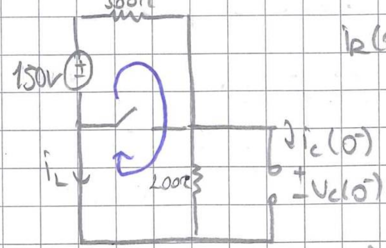
$V_3(\infty) = 0 \rightarrow$ no voltage on capacitor.

*(3) $V_C(0^+) = V_C(0^-)$, obtain $t = 0^+$

$V_1 e^0 + V_2 e^0 + 0 = V_C(0^+)$

$V_1 + V_2 = V_C(0^+)$

*(4) $V_C(0^-)$, $t < 0$



$i_R(0^-) = -i_L(0^-)$

$i_R(0^-) = \frac{150}{300} = 0{,}3A$

$i_L(0^-) = i_L(0^+) = +0{,}3A$

$V_C(0^-) = V_C(0^+) = 200\Omega \times 0{,}3A = 60V$

goes on

⑤ when we find $V_c(0^+)$ then,

$V_c(0^+) = V_1 e^0 + V_2 e^0$

$\underbrace{60 = V_1 + V_2}$ (our 1st eq)

⑥ if we take derivative

$\frac{dV_c(t)}{dt} = V_1 S_1 e^{s_1 t} + V_2 S_2 e^{s_2 t}$

@ $t = 0^+ \rightarrow \frac{dV_c(0^+)}{dt} = \frac{I_c(0^+)}{C}$

$S_1 = -\alpha + \sqrt{\alpha^2 - w_0^2} = -50000$
$S_2 = -\alpha - \sqrt{\alpha^2 - w_0^2} = -200000$

⑦ we need to find $I_c(0^+)$, use KCL @ upper node at $t > 0^+$

$i_c(0^+) + i_L(0^+) + i_R(0^+) = 0$

$i_c(0^+) = -i_L(0^+) - i_R(0^+)$
$= -(-0,3) - \left(\frac{V_R(0^+)}{R}\right) \rightarrow V_R(0^+) = V_c(0^+)$
$= 0,3 - \frac{60}{200} = \underline{\underline{0A}}$

when we put $I_c(0^+)$,

$\Rightarrow \frac{dV_c(0^+)}{dt} = 0 \rightarrow \frac{dV_c(0^+)}{dt} = V_1 S_1 e^{S_1 (t \to 0)} + V_2 S_2 e^{(t \to 0)}$

$\frac{dV_c(0^+)}{dt} = V_1 S_1 + V_2 S_2$

$0 = V_1(-50000) + V_2(-200000)$

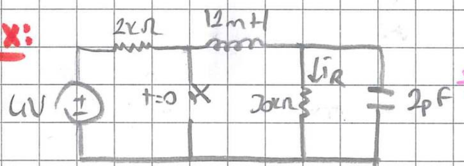
$\underline{0 = V_1 + 4V_2}$ (2nd eq)

⑧ if we solve 1st and 2nd equation,

$\left.\begin{matrix} 60 = V_1 + V_2 \\ 0 = V_1 + 4V_2 \end{matrix}\right\}$ $\underline{V_1 = 80V}$, $\underline{V_2 = -20V}$

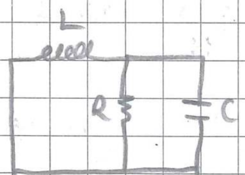
$V_c(t) = 80 e^{-50000t} - 20 e^{-200000t}$ @ $t > 0$

Ex:



Find the i_R for all time intervals

① $t > 0$,



$\alpha = \frac{1}{2RC} = 8,33 \cdot 10^6 s^{-1}$

$w_0 = \frac{1}{\sqrt{LC}} = 6.45 \cdot 10^6 \text{ rad } s^{-1}$

$\underbrace{\alpha > w_0}$

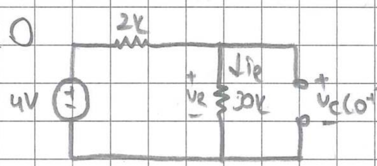
$i_R(t) = \underbrace{I_1 e^{s_1 t} + I_2 e^{s_2 t}}_{\text{transient}} + \underbrace{I_3}_{\text{steady}}$

$S_1 = -\alpha + \sqrt{\alpha^2 - w_0^2} = -3,06 \cdot 10^6 s^{-1}$
$S_2 = -\alpha - \sqrt{\alpha^2 - w_0^2} = -13,6 \cdot 10^6 s^{-1}$

$I_3 = 0 \rightarrow$ we have no energy

$i_R(t) = I_1 e^{-3,06 \cdot 10^6 t} + I_2 e^{-13,6 \cdot 10^6 t}$

② $t < 0$



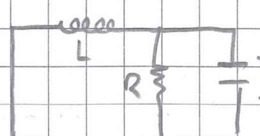
$i_L(0^-) = i_R(0^-) = \frac{4V}{32k\Omega} = 125 \mu A$

$V_c(0^-) = i_R(0^-) \times 30k\Omega$
$= 3,75V$

$i_L(0^-) = i_L(0^+) = 125 \mu A$
$V_c(0^-) = V_c(0^+) = 3,75V$

$i_R(0^+) = \frac{V_R(0^+)}{30k\Omega} = 125 \mu A$

③ $t = 0^+$



$V_L = L \frac{di_L}{dt}$ $V_R = V_c$

$\underline{\frac{I_c(0^+)}{RC} = \frac{di_R}{dt}}$

$I_1 e^0 + I_2 e^0 + 0 = i_R(t)$

$\underline{I_1 + I_2 = 125 \cdot 10^{-6} A}$ ①

KCL @ upper node

$i_L(0^+) = i_R(0^+) + i_c(0^+) = 0$

$\frac{di_R}{dt} = 0$

$I_1 s_1 + I_2 s_2 = 0$

$-3{,}06\cdot10^6 I_1 - 13{,}6\cdot10^6 I_2 = 0$ ②

* ④ If we solve together 1 & 2

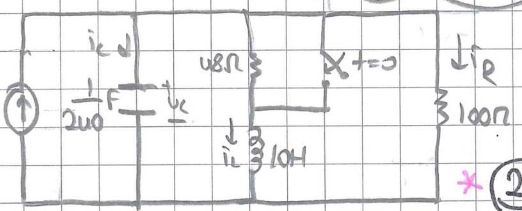
$I_1 = 161{,}3\,\mu A, \quad I_2 = -36{,}3\,\mu A$

$I_R(t) = (161{,}3e^{-3{,}06\cdot10^6 t} - 36{,}3e^{-13{,}6\cdot10^6 t})\,\mu A, \quad t>0$

$I_R(t) = 125\,\mu A, \quad t<0$

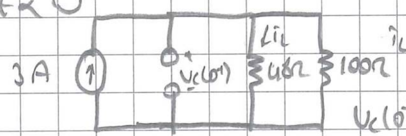
Ex:

$3u(-t)A$ — we have a source before $t=0$



determine $i_L(t)$ for the circuits and plot the wave form of your solution.

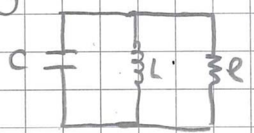
* ② $t<0$



$i_L(0^-) = 3A \cdot \frac{100\Omega}{148\Omega} = 2{,}02A$

$V_C(0^-) = 48\cdot 2{,}02 = 97{,}3V$

* ① $t>0$



$\alpha = 1/2RC = 1{,}2\,s^{-1}$

$w_0 = 1/\sqrt{LC} = 4{,}9\,rad\,s^{-1}$

$\alpha < w_0 \rightarrow$ under-damped

$i_L(t) = e^{-\alpha t}[I_1\cos(w_d t) + I_2\sin(w_d t)] + I_3$

$w_d = \sqrt{w_0^2 - \alpha^2} = 4{,}75\,rad\,s^{-1}$

$I_3 = 0$ (we don't have source @ $t>0$)

* ③ $t=0^+$, $i_L(0^+) = i_L(0^-)$

$i_L(t) = e^0[I_1\cos(0) + I_2\sin(0)] + 0$

$2{,}02A = I_1$

* ④ $\frac{d\,i_L(0^+)}{dt} = ?$ $\quad V_L = L\frac{di_L}{dt}$

$\frac{V_L(0^+)}{L} = \frac{di_L(0^+)}{dt} \quad V_L(0^+) = V_C(0^+) = 97{,}3V$

$\frac{di_L(0^+)}{dt} = \frac{97{,}3}{10} = 9{,}73A$

* ⑤ $\frac{di_L}{dt} = -\alpha e^{-\alpha t}[I_1\cos(w_d t) + I_2\sin(w_d t)] + [-w_d I_1\sin(w_d t) + w_d I_2\cos(w_d t)]e^{-\alpha t}$

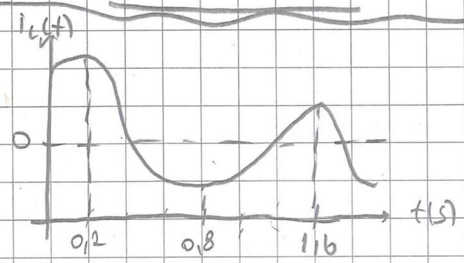
@ $t=0$

$9{,}73 = (-1{,}2)(2{,}02) + 4{,}75 I_2$

$I_2 = 2{,}56A$

$i_L(t) = e^{-1{,}2t}[2{,}02\cos(4{,}75t) + 2{,}56\sin(4{,}75t)]\,A, \quad t>0$

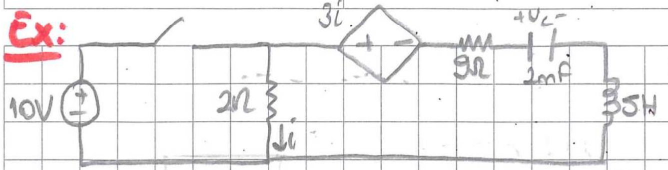
$i_L(t) = 2{,}02\,A, \quad t<0$



$4{,}75 = 2\pi f$

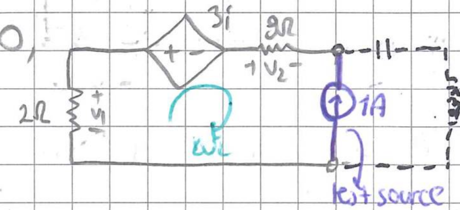
$f = 0{,}75 \rightarrow T = 1/f = 1{,}33\,sec$

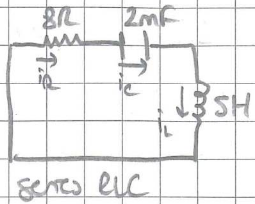
Ex:



Find an expression for $V_c(t)$ for $t>0$.

① $t>0$,





$\alpha = \dfrac{R}{2L} = 0{,}8\,s^{-1}$

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 10\,rad\,s^{-1}$

$\alpha < \omega_0 \rightarrow$ under-damped

we need to apply 4th approach to find $R_{eq} = R_{th}$

$-V_1 + 3i + V_2 + V_T = 0$

$-2i + 3i - 9i + V_T = 0$

$V_T = 8i$

$V_T = 8 \cdot 1A = 8V$

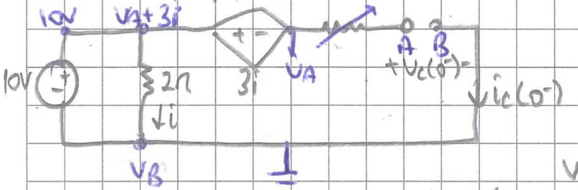
$R_{eq} = \dfrac{V_T}{1A} = 8\Omega$

$V_c(t) = e^{-\alpha t}\left[V_1 \cos(\omega_d t) + V_2 \sin(\omega_d t)\right] + V_3$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 9{,}97\,rad\,s^{-1}$

@ $t \to \infty$, $V_3 = 0$ because we have no source.

② $t<0$, for find $V_c(0^+)$



$i_c(0^-) = i_c(0^+) = 0A$ for open circuit

$i = \dfrac{10V}{2\Omega} = 5A$

$V_c(0^-) = V_c(0^+) = V_A - V_B \quad \rightarrow V_A + 3i = 10V$

$V_c(0^-) = V_c(0^+) = -5V \quad \begin{cases} V_A = 10V - 15V = -5V \\ V_B = 0V \end{cases}$

③ $t = 0^+$

$-5V = e^0\left[V_1 \cos 0 + V_2 \sin 0\right] + 0$

$\underline{-5V = V_1}$

④ $\dfrac{dV_c(0^+)}{dt} \rightarrow i_c(t) = C\dfrac{dV_c(t)}{dt}$

$\dfrac{dV_c(0^+)}{dt} = i_c(0^+) \cdot C = 0$

we take derivative and put α, V_1 $\rightarrow 0 = -0{,}8(-5V) + 9{,}97 V_2$

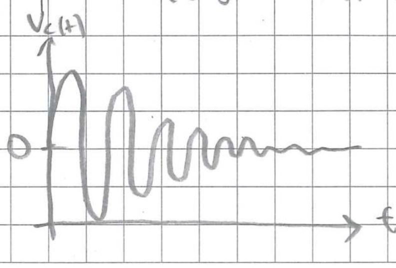
$\underline{V_2 = -0{,}4V}$

⑤ $V_c(t) = e^{-\alpha t}\left[V_1 \cos(\omega_d t) + V_2 \sin(\omega_d t)\right] + V_3$

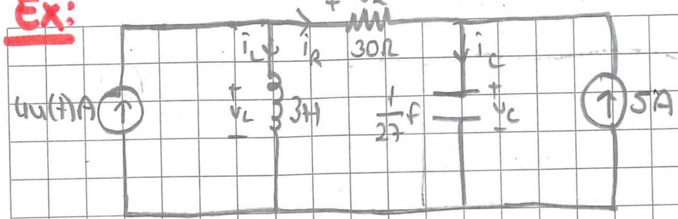
$V_c(t) = e^{-0{,}8t}\left[-5\cos(9{,}97t) - 0{,}4\sin(9{,}97t)\right], \quad t>0$

$V_c(t) = -5V, \quad t<0$

* for $\alpha < \omega_0$ and our circuit is under-damped, we have ossilation,

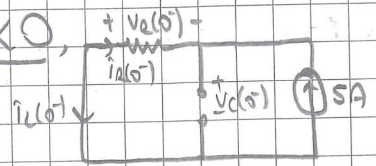


Ex:



Find i_R, i_L, i_c and V_R, V_L, V_c at $t>0$ and $t<0$.

* $t<0$,



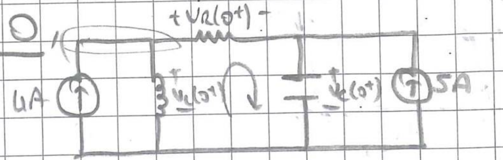
$V_R(0^-) = (30\Omega)(-5A) = -150V$ $\quad i_R(0^-) = -5A$

$V_L(0^-) = 0V$ $\quad i_L(0^-) = 5A$

$V_c(0^-) = 150V$ $\quad i_c(0^-) = 0A$

* $V_c(0^-) = V_c(0^+)$ and $i_L(0^-) = i_L(0^+)$

* $t>0$,



KCL @ upper left

$4A = 5A + i_R(0^+)$

$i_R(0^+) = -1A$

KCL @ upper right

$i_R(0^+) + 5A = i_c(0^+)$

$i_c(0^+) = 4A$

$V_R(0^+) = (30\Omega)(-1A) = -30V$

$V_c(0^+) = 150V$ $\quad i_L(0^+) = 5A$

for find $V_L(0^+)$ → $-V_L + V_R + V_c = 0$

$V_L(0^+) = 120V$

we can find derivatives at $t>0$,

$\frac{d\,i_L(0^+)}{dt} \rightarrow V_L = L\frac{d\,i_L}{dt}$

$\frac{d\,i_L(0^+)}{dt} = \frac{V_L(0^+)}{dt} \Rightarrow \frac{120V}{3H} = 40A/s$

$\frac{d\,i_c(0^+)}{dt} \rightarrow i_c = C\frac{dV_c}{dt}$

$\frac{d\,i_c(0^+)}{dt} = \frac{dV_c(0^+)}{dt} \Rightarrow \frac{4A}{1/27} = 108V/s$

$\frac{dV_R(0^+)}{dt} \rightarrow V = I \cdot R$

$\frac{dV_R(0^+)}{dt} = \frac{d\,i_R(0^+)}{dt} \cdot R = (-40A/s)(30) = -1200V/s$

$\frac{d\,i_R(0^+)}{dt} \rightarrow 4 - i_L - i_R = 0$

$\frac{d\,i_R(0^+)}{dt} = 0 - \frac{d\,i_L(0^+)}{dt} = -40A/s$

$\frac{d\,i_c(0^+)}{dt} \rightarrow 5 + i_R = i_c$

$0 + \frac{d\,i_R(0^+)}{dt} = \frac{d\,i_c(0^+)}{dt} = -40A/s$

$\frac{dV_L(0^+)}{dt} \rightarrow -V_L + V_R + V_c = 0$

$\frac{dV_L(0^+)}{dt} = \frac{dV_R(0^+)}{dt} + \frac{dV_c(0^+)}{dt}$

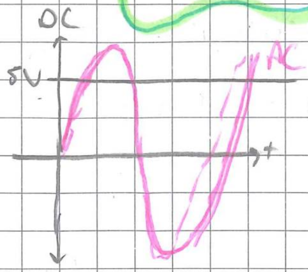
$\frac{dV_L(0^+)}{dt} = -1092V/s$

$\frac{d\,i_R(0^+)}{dt} = -40A/s$ $\quad \frac{dV_R(0^+)}{dt} = -1200V/s$

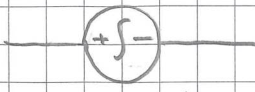
$\frac{d\,i_L(0^+)}{dt} = 40A/s$ $\quad \frac{dV_L(0^+)}{dt} = -1092V/s$

$\frac{d\,i_c(0^+)}{dt} = -40A/s$ $\quad \frac{dV_c(0^+)}{dt} = 108V/s$

* ALTERNATE CURRENT (AC) *



we can show that in circuits



AC notation

$$V(t) = V_A \sin(\omega t + \phi) = V_A \cos(\omega t + \phi + \pi/2)\ V$$

V_A → magnitude

ω → angular freq.

ϕ → phase

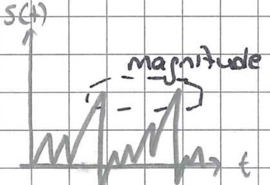
$\omega = 2\pi f$

$f = \frac{1}{T}$ → period

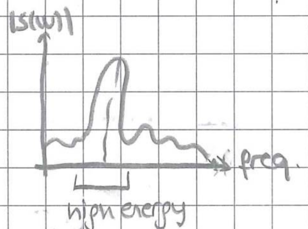
fourier transform → any signal can be transform to sum of the sinusoids.

$$v(t) = \sum_{n=-\infty}^{\infty} A_n \cos(\omega_n t + \theta_n) \quad , \omega_n = 2\pi f_n$$

* In AC analysis, your capacitor and inductor behaviors are charging by the frequency.



fourier →

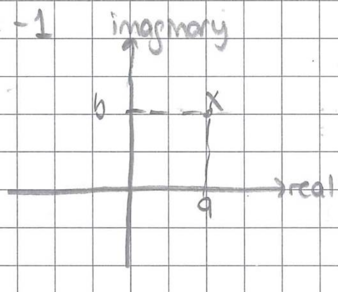


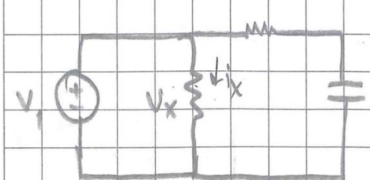
* input → output, magnitude and phase can be changed but the frequency must be the same.

→ * Complex Numbers: $j^2 = -1$

$x = a + bj$

$\underbrace{a = Re\{x\} \ , \ b = Im\{x\}}_{\text{rectangular form}}$





$i_x = I_x \cos(2\pi f t + \phi)$

$V_1 = A\cos(2\pi f t + \phi) \rightarrow V_x = \underbrace{A_x \cos(2\pi f t + \phi)}$

$Re = \{A e^{j(2\pi f t + \phi)}$

↓

$A e^{j\phi} = A\cos\phi + jA\sin\phi$ (euler's)

$e^{j\phi} = \cos\phi + j\sin\phi \rightarrow |z|e^{j\phi} = a + bj$

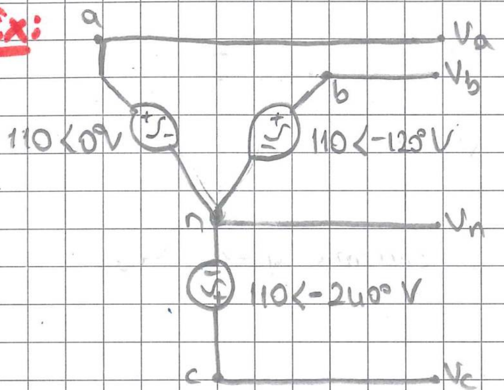
$a = |z|\cos\phi$
$b = |z|\sin\phi$ $\Big]$ $|z| = \sqrt{a^2+b^2}$
$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$|z| \rightarrow$ magnitude of z

$\phi \rightarrow$ phase/angel of z

$$* \quad \underbrace{z = a+bj}_{\text{rectangular form}} = \underbrace{|z|e^{j\phi}}_{\text{exponential form}} = \underbrace{|z|\angle\phi}_{\text{polar form}} \quad !$$

Ex:



Determine the quantity $V_a - V_b$ in polar form if $V_n = 0$.

$V_a - V_b = 110\angle 0° - 110\angle -120°$

$= [110\cos 0 + j110\sin 0] - [110\cos(-120) + j110\sin(-120)]$

$= [110 + j0] - [110(-1/2) + j110(-\sqrt{3}/2)]$

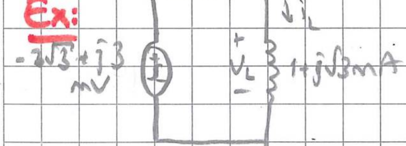
$= 165 + j55\sqrt{3}$

$|V_a - V_b| = \sqrt{(165)^2 + (55\sqrt{3})^2} \angle \tan^{-1}(55\sqrt{3}/165)$

$= \underline{190.5\angle 30°}$

$$* \quad \left.\begin{array}{l} z_1 = a_1 + b_1 j = |z_1| e^{j\theta_1} \\ z_2 = a_2 + b_2 j = |z_2| e^{j\theta_2} \end{array}\right\} \quad \left.\begin{array}{l} z_1 z_2 = |z_1||z_2| \angle \theta_1 + \theta_2 \\ \dfrac{z_1}{z_2} = \dfrac{|z_1|}{|z_2|} \angle \theta_1 - \theta_2 \end{array}\right\} \; !$$

Ex:

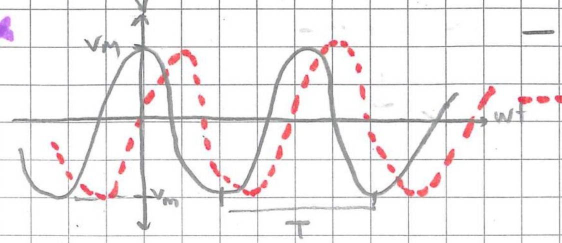


determine the ratio of V_L to i_L

$\frac{V_L}{i_L} = \frac{|V_L|}{|i_L|} \angle \theta_{V_L} - \theta_{i_L}$

$= \frac{-3\sqrt{3} + j3 \text{ mV}}{1 + j\sqrt{3} \text{ mA}} = \frac{\sqrt{(-3\sqrt{3})^2 + (3)^2} \angle \tan^{-1}(1/-\sqrt{3})}{\sqrt{1^2 + (\sqrt{3})^2} \angle \tan(\sqrt{3})} = \frac{6\angle 150°}{2\angle 60°} = \underline{3\angle 90°}$

*



— $v_1 = V_m \sin(\omega t)$

--- $v_2 = V_m \sin(\omega t + \theta)$

$\}$ v_1 leads v_2 by θ
v_2 lags v_1 by θ

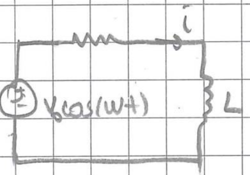
$\omega = 2\pi f \quad T = \frac{1}{f}$

→ to find phase difference (θ);

$t_s = 0,2s$ (phase diff)
$f_s = 0,5Hz$
$T = 1/0,5 = 2\ s$ ← periode

1 cycle 2 sec 2π
0,2sec ?

$? = 0,2\pi = \theta_s$.



$$\frac{di(t)}{dt} + \frac{R}{L} i(t) = \frac{V_0}{L}\cos(wt)$$ → ossilates forever

$$i(t) = I_0 \cos(wt + \theta)$$

$$i(t) = \frac{V_0}{\sqrt{R^2 + w^2L^2}} \cos\left\{wt - \tan^{-1}\left(\frac{wL}{R}\right)\right\}$$

* Frequency (w) is not changing.

* The RC / RL circuit's transient response is negligible after 5τ.

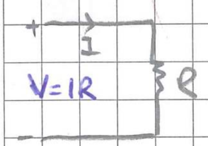
* The RLC circuit's transient response is negligible after $T = t_s$.

* The remaining response is sinusoidals.

$$v(t) = V_M \cos(wt + \phi_1), \quad i(t) = I_M \cos(wt + \phi_2)$$

$v_R(t) = I \cdot R$
$\phi_1 = \phi_2$

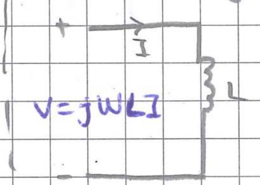
→ V and I are in phase



$v_L(t) = L\dfrac{di(t)}{dt}$

$\dfrac{V_M}{I_M} = wL$
$\phi_1 = \phi_2 + 90°$

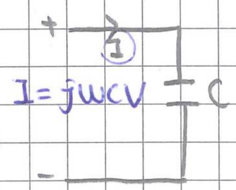
→ V leads I by 90°



$i_C(t) = C\dfrac{dv_C(t)}{dt}$

$\dfrac{I_M}{V_M} = wC$
$\phi_2 = \phi_1 + 90°$

→ I leads V by 90°



$$v_L = Ljw\frac{Ae^{j(wt+\theta)}}{I_L} \rightarrow \frac{V_L}{I_L} = wL = Z = \text{impedance}.$$

*** Z (impedance):**

$Z_R = R$

$Z_L = jwL$

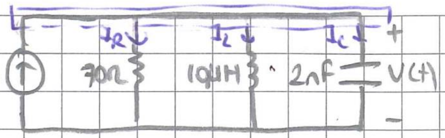
$Z_C = \dfrac{1}{jwC} \rightarrow -j\left(\dfrac{1}{wC}\right)$

	High frequency	Low frequency
Inductor	large Z	small Z
Capacitor	small Z	large Z

* passive filters → R, L, C
  active filters → Op-Amps

Ex:

$\overbrace{8\cos(\underbrace{2\pi 10^6}_{w} t + 30^\circ)}^{}\,mA$ → $8\angle 30^\circ$



Determine V(t).

$I_s(t) = 8\angle 30^\circ$

$Z_R = 70 \quad , \quad Z_L = jwL = j\,2\pi \cdot 10^6 \cdot 10\,10^{-6} = j20\pi$

$Z_C = \dfrac{1}{jwc} = \dfrac{1}{j2\pi \cdot 10^6 \cdot 2\cdot 10^{-9}} = \dfrac{10^3}{-j4\pi}$

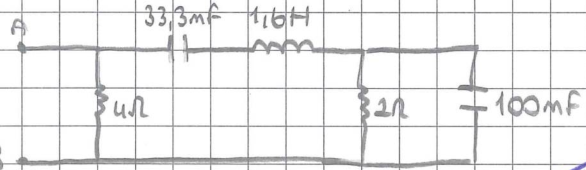
$KVL \rightarrow 8\angle 30^\circ = \dfrac{V}{70} + \dfrac{V}{j20\pi} + \dfrac{Vj4\pi}{10^3}$

$8\angle 30^\circ = V\left[\dfrac{1}{70} + \dfrac{1}{j62{,}8} + j(0{,}012)\right]$

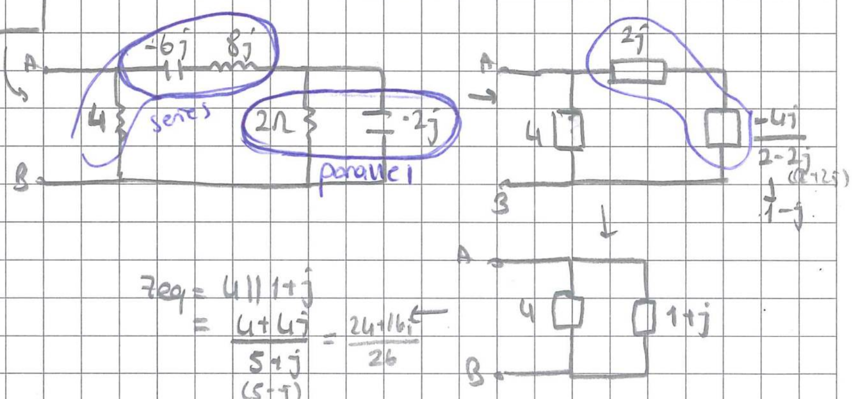
$8\angle 30^\circ = V\,(\underbrace{0{,}0143 - 0{,}0033j}_{0{,}0147\angle -13^\circ}) \rightarrow V = \dfrac{8\angle 30^\circ}{0{,}0147\angle -13^\circ} = 545\angle 43^\circ\, mV$

$\underline{V(t) = 545\cos(2\pi 10^6 t + 43^\circ)\, mV}$

Ex:



Determine the equivalent impedance of the network at terminals A-B if $W = 5\,rads^{-1}$



$Z_L = jwL = j\,5(1{,}6) = 8j$

$Z_{C_1} = \dfrac{1}{jwc} = \dfrac{1}{j5(33{,}3\cdot 10^{-3})} = -6j$

$Z_{C_2} = \dfrac{1}{j5(100\cdot 10^{-3})} = -2j$

$Z_{eq} = 4 \parallel 1+j$

$= \dfrac{4+4j}{5+j}\,(5-j) = \dfrac{24+16j}{26}$

$= 0{,}92 + j0{,}62 \rightarrow \underline{Z_{eq} = 1{,}1\angle 33{,}9^\circ}$

* Reactance *

$Z = \dfrac{V}{I} = R + j\,X$ imaginary part

$Z_R = R \rightarrow X = 0$

$Z_L = jwL \rightarrow X = wL$

$Z_C = \dfrac{1}{jwc} \rightarrow X = \dfrac{1}{wc}$

* Admittance *

$Y = \dfrac{1}{Z}$

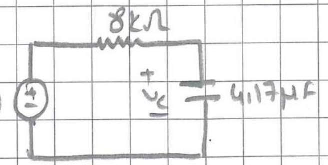
$Y_R = \dfrac{1}{R}$

$Y_L = \dfrac{-j}{wL}$

$Y_C = jwc$

NOTE: If we have two sources with two different frequency, we need to apply "superposition" theorem.

Ex:

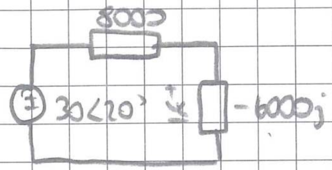


Determine $V_c(t)$ on time domain.

$30\cos(40t+20°) \rightarrow 30\angle 20°$

$Z_R = 8000$

$Z_C = \frac{1}{j\cdot 40 \cdot 4{,}17\cdot 10^{-6}} = -6000j$



$V_c = \frac{(30\angle 20°)(-6000j)}{8000-6000j}$

$= \frac{(30\angle 20°)(6000\angle -90°)}{(10000\angle -37°)}$

$V_c = 18\angle -33°$

$\underline{V_c(t) = 18\cos(40t-33°)}$